Labor Market: Job Search and Unemployment

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Roadmap

1. Introduction

2. Basic job search model

3. Empirics of job search

Another Nobel Prize field

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2010

Peter A. Diamond

Prize share: 1/3

Dale T. Mortensen

Prize share: 1/3

Christopher A. Pissarides

Prize share: 1/3

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2010 was awarded jointly to Peter A. Diamond, Dale T. Mortensen and Christopher A. Pissarides "for their analysis of markets with search frictions"

What's it about?

Search: frictions in matching

- It takes time (and effort and perhaps other resources) to meet a partner and to learn the uncertain value of any partnership
- These lags are called ”frictions” in the market, and acknowledged in the search theories
- With such frictions, the matching market becomes fluid and fledged: meet one, accept or reject, stay alone, meet another, exogenous or endogenous breakup, search effort, search while staying in a match, mismatch, posting, bargaining, commit, ...

Search models of labor market

- An advanced topic in labor economics and macroeconomics
  - Dynamic models with uncertainty
  - Pair-wise market instead of a centralized market and a single price

- Many elements we have learned can be embedded into search models
  - Human capital investment
  - Learning from signals
  - Labor supply and family
  - Labor market power
  - Compensating differential
  - Sorting

- Important topics in this field
  - Unemployment, on-the-job search, job ladder, worker mobility, labor market dynamics, wage bargaining, outside option, job contracts, income differences, match quality, matching efficiency, business cycle, wage rigidity, job loss, search channels, ...
  - Unemployment insurance, labor protection and regulation, labor market institutions, outsourcing, temporary agency, ...

Today's plan

- Show you the simplest sequential (random) job search models
  - We focus on a discrete-time version to ease understanding
  - Without talking about any math details of dynamic programming
  - Show a continuous-time version (which is most often used in real studies given its analytic convenience) in the appendix
  - The model is partial equilibrium—we abstract from the firm side
  - There is another strand of framework in the literature called directed search models, which is substantially more technical
- Although the math is more advanced, the intuition is still simple
  - One perhaps useful analogy: think that you have a hen that, in each day, lays an egg and can be replaced with another hen randomly drawn, and that different hens lay eggs of different sizes (values)
- Bring the model to the data by showing you the empirics in some recent studies
  - While the models in these studies generally equip with more sophisticated extensions and features to fit data, the intuitions gained from our basic model largely retain
  - They also give a brief preview on how job search models can be interconnected with some other topics that we have learned

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Basic model of sequential job search

- Consider an individual searching for a job in discrete time, seeking to maximize expected present discounted value of lifetime income/consumption $\mathbb{E}\Sigma_{t=0}^{\infty}\beta^t c_t$
  - Income is $c = w$ if employed at wage w and $c = b$ if unemployed (unemployment benefit or value of leisure or home production)

- At each time period, the unemployed individual samples one job offer (i.i.d. draw; that's why we call it "random search"), and decides whether to take or continue searching
  - The offered wage is from an exogenous, stationary, and known distribution $F(w)$
  - Thus undirected (random) search model: individual has no ability to seek or direct his search towards different parts of the wage distribution (or towards different types of jobs)

- Assume no recall and for now if a job is accepted he/she will be employed at that job forever
  - The present discounted value of accepting a job with wage w is thus $W(w) = \Sigma_{t=0}^{\infty}\beta^t w = w + \beta W(w) \Rightarrow W(w) = w/(1-\beta)$

Bellman equation and reservation wage

- The present discounted value of the individual after a draw in the beginning of the period satisfies a Bellman equation:
  $V(w) = \max\left\{\frac{w}{1-\beta}, b + \beta \int V(\omega) dF(\omega)\right\}$ ($\equiv \max\{W(w), U\}$)
  - First term gives the value of accepting the offer
  - Second term gives the value of turning down (and unemployment), which can be also defined recursively using a Bellman equation:
    $U = b + \beta \int V(w) dF(w) = b + \beta \int \max\{\frac{w}{1-\beta}, U\} dF(w)$
  - Thus we can write $V = T(V)$ and solve the value function V (and similarly for U) using a contraction mapping (see python code)

- Since $\frac{w}{1-\beta}$ is strictly increasing in w, it's easy to see that the optimal policy will take a reservation wage form: there will exist some reservation wage R such that $\frac{R}{1-\beta} = U$
  - Forr all $w \geq R$: $V(w) = \frac{w}{1-\beta}$
  - For all $w < R$: $V(w) = U = \frac{R}{1-\beta}$
  - $V \equiv \int V(\omega) dF(\omega) = \frac{RF(R)}{1-\beta} + \int_{w \geq R} \frac{w}{1-\beta} dF(w)$
  - $R = (1-\beta)b + \beta \int \max\{w, R\} dF(w) = T(R)$

How unemployment insurance (UI) or wage distribution affects reservation wages and unemployment

- With some tedious algebra ‣ see derivation, we get
  $R - b = \beta(Ew - b) + \beta \int_{w \leq R} F(w) dw$
  - $Ew = \int w dF(w)$ is the mean of the wage distribution
  - Note that both LHS and RHS increase in R

- An increase in b (more generous UI) increases reservation wage R and thus unemployment ratio (see next slide)

- A mean preserving spread of F to $\tilde{F}$ (more risker draws but keeping the mean same ‣ techniques) increases R and unemployment ratio
  - Greater option value of waiting when faced with a more dispersed wage distribution: lower wages are already turned down, while higher wages are now more likely

- Alternatively this notebook shows that you can solve the model numerically and then examine how R depends on different parameters by simulation ‣ see one result

Unemployment under sequential search

- Suppose a continuum 1 of identical individuals sampling jobs from same stationary distribution F
  - Once a job is created, it lasts until the worker dies, with probability s
  - A mass of s workers born every period (as unemployed), so that population is constant
  - This die-reborn setting is just a trick: more natural to think layoff risk
- Now effective discount factor of workers is $\tilde{\beta} \equiv \beta(1-s)$, and value of accept: $W(w) = \frac{w}{1-\tilde{\beta}}$; Reservation wage as $\frac{R}{1-\tilde{\beta}} = U$
- Law of motion of unemployment:
  - Start time t with u_t unemployed workers
  - $\Rightarrow u_{t+1} = s + (1-s)F(R)u_t$
  - $\Rightarrow u_{t+1} - u_t = \underbrace{s(1-u_t)}_{\text{flow in / job destruction}} - \underbrace{(1-s)(1-F(R))u_t}_{\text{flow out / job creation}}$
- Steady-state (constant) unemployment rate: $u = \frac{s}{s+(1-s)(1-F(R))}$
  - $u > s$ since $F(R) > 0$; An increase in R (a higher reservation wage) will depress job creation and increase unemployment

Arriving rate and search intensity

- Now we assume that at each period, the new job offer arrives only with probability α
- $U = b + \alpha\beta \int \max\{W(w), U\} dF(w) + (1-\alpha)\beta U$
  - Note $W(w)$ does not change and we still have $W(R) = U$
  - When $\alpha = 1$, we return back to the basic model (Q: how does the value of α affect R?)
- Suppose a worker can affect the arrival rate of offers α, at cost $g(\alpha)$, where $g' > 0$ and $g'' > 0$
- Unemployed workers choose α to maximize U:
  $U = \max_\alpha b - g(\alpha) + \alpha\beta \int \max\{W(w), U\} dF(w) + (1-\alpha)\beta U$
- The FOC for an interior solution is
  $\beta \int \max\{W(w), U\} dF(w) - \beta U = g'(\alpha)$
  - Note the LHS can be rewritten as $\beta \int \max\{\frac{w-R}{1-\beta}, 0\} dF(w)$
- Recall that an increase in b will raise R and hence will reduce the search intensity α

On-the-job search

- Suppose now a new offer arrives in each period with exogenous probability α_0 while unemployed and α_1 while employed
- $U = b + \alpha_0 \beta \int \max\{W(w), U\} dF(w) + (1 - \alpha_0)\beta U$;
- $W(w) =$
  $w + \alpha_1 \beta \int \max\{W(w'), W(w)\} dF(w') + (1 - \alpha_1)\beta W(w)$
- As W is still increasing in w, unemployed workers use a reservation wage satisfying $W(R) = U$, and employed workers switch jobs whenever $w' > w$ (and thus climb a "job ladder")
- Combine two equations at $w = R$, we have:
  $R = b + (\alpha_0 - \alpha_1) \beta \int \max\{W(w') - W(R), 0\} dF(w')$
  - Note $R < b$ if $\alpha_0 < \alpha_1$: if a worker gets offers more frequently when employed, he is willing to accept wages below b
- The model (with layoff risks) can analyze individual transitions between U and E, and between E, and makes predictions
  - Employed span is positively correlated with wage
  - Wage and tenure is negatively correlated with separation rates

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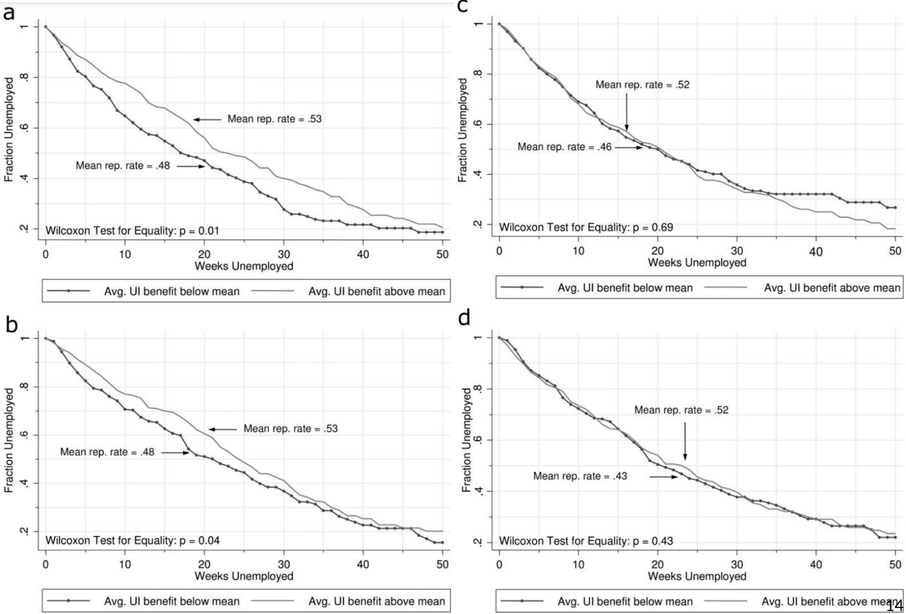
Job search and unemployment insurance

- One of the classic theoretical results of job search model (and empirical results in public finance) is that unemployment insurance (UI) raises unemployment and reduces labor supply
  - This finding has traditionally been interpreted as evidence of moral hazard caused by a substitution effect: UI distorts the relative price of leisure and consumption, reducing the marginal incentive to search for a job, and subsidizing unproductive leisure
- Chetty (2008) questions whether it is not only due to the substitution effect but also due to a ”liquidity effect”
  - It is motivated by the observation that many unemployed individuals have limited liquidity and exhibit excess sensitivity of consumption to cash on hand (i.e. high marginal propensities to consume due to high marginal utility of consumption under unbounded FOC)
- In a job search model with asset holding and incomplete credit and insurance markets, UI benefits increase cash on hand and consumption for a hand-to-mouth agent while unemployed, who will thus face less pressure to find a new job quickly, leading to a longer unemployment duration

UI impact on unemployment duration by wealth quartile



Job search behavior: unemployed vs. employed

- Faberman et al. (2022) provide the most comprehensive evidence to date on the nature of on-the-job search in US

TABLE I

BASIC JOB SEARCH STATISTICS BY LABOR FORCE STATUS.

	Employed	Unemployed	Out of Labor Force
Percent that actively searched for work	22.4 (0.7)	99.6 (0.8)	2.4 (0.6)
Percent that actively searched and are available for work	13.2 (0.6)	99.6 (0.5)	0.0 (0.0)
Percent reporting no active search or availability, but would take job if offered	5.9 (0.4)	0.2 (0.3)	6.1 (0.9)
Percent applying to at least one vacancy in last four weeks	21.4 (0.7)	92.8 (1.7)	2.2 (0.6)
Percent with positive time spent searching in last seven days	21.3 (0.7)	86.7 (2.3)	2.3 (0.6)
Conditional on Active Search			
Percent only searching for an additional job	36.0 (1.7)	—	—
Percent only seeking part-time work	21.7 (1.5)	22.5 (2.8)	50.9 (12.9)
Percent only seeking similar work (to most recent job)	25.3 (1.7)	7.4 (1.8)	33.7 (14.3)
N	3725	228	706

Note: Estimates come from authors' tabulations from the October 2013–2017 waves of the SCE Job Search Supplement, for all individuals aged 18–64, by labor force status. Standard errors are in parentheses.

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On-the-job search effort declines in current wage

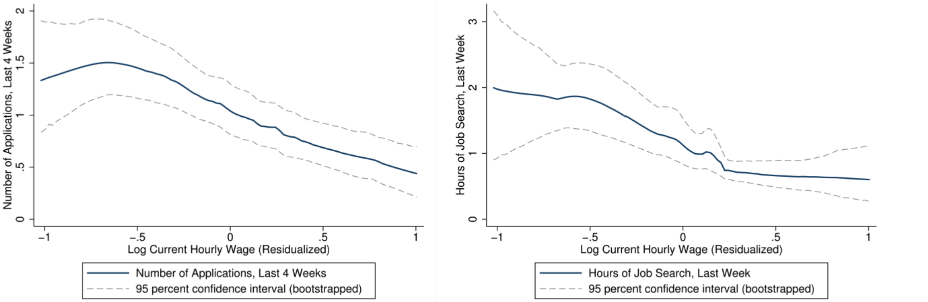


FIGURE 2.—Job search effort by the current wage. *Notes*: Figure reports the LOWESS estimates (with smoothing parameter 0.8) of the relationship between the measures of search effort listed on each vertical axis and the (log) real current wage of the employed, residualized after controlling for observable worker characteristics (see Table III for the list of specific variables). Dashed lines represent 95 percent confidence intervals. The confidence intervals are based on a bootstrap with 500 replications. The estimates use all employed individuals, excluding the self-employed, aged 18–64 from the October 2013–2017 waves of the SCE Job Search Supplement.

On-the-job search is more effective

TABLE IV

JOB SEARCH OUTCOMES BY LABOR FORCE STATUS.

	Employed			Unemployed	Out of Labor Force
	Looking for Work	Not Looking	All		
A. All Search Outcomes					
Mean offers	0.417	0.115	0.191	0.666	0.130
	(0.033)	(0.023)	(0.019)	(0.286)	(0.024)
Fraction with at least one formal offer, including unsolicited offers	0.261	0.053	0.105	0.342	0.078
	(0.015)	(0.004)	(0.005)	(0.037)	(0.010)
Fraction with at least one unsolicited offer	0.044	0.025	0.030	0.042	0.033
	(0.007)	(0.003)	(0.003)	(0.016)	(0.007)
Fraction with at least one formal or unrealized offer	0.318	0.094	0.150	0.370	0.089
	(0.016)	(0.006)	(0.006)	(0.038)	(0.011)
Fraction of best formal offers accepted	0.460	0.111	0.328	0.493	0.195
	(0.036)	(0.027)	(0.027)	(0.069)	(0.052)
B. Ignoring Search Outcomes for Additional Jobs					
Mean offers	0.258	0.111	0.148	0.666	0.130
	(0.024)	(0.023)	(0.018)	(0.286)	(0.024)
Fraction with at least one formal offer, including unsolicited offers	0.173	0.051	0.081	0.342	0.079
	(0.013)	(0.004)	(0.005)	(0.037)	(0.010)
Fraction with at least one unsolicited offer	0.030	0.024	0.026	0.042	0.033
	(0.006)	(0.003)	(0.003)	(0.016)	(0.007)
Fraction with at least one formal or unrealized offer	0.217	0.091	0.123	0.370	0.089
	(0.014)	(0.006)	(0.006)	(0.038)	(0.011)
Fraction of best formal offers accepted	0.488	0.106	0.309	0.493	0.195
	(0.046)	(0.028)	(0.030)	(0.069)	(0.052)

On-the-job search targets better jobs

TABLE VI

CHARACTERISTICS OF BEST JOB OFFER BY LABOR FORCE STATUS AT TIME OF OFFER.

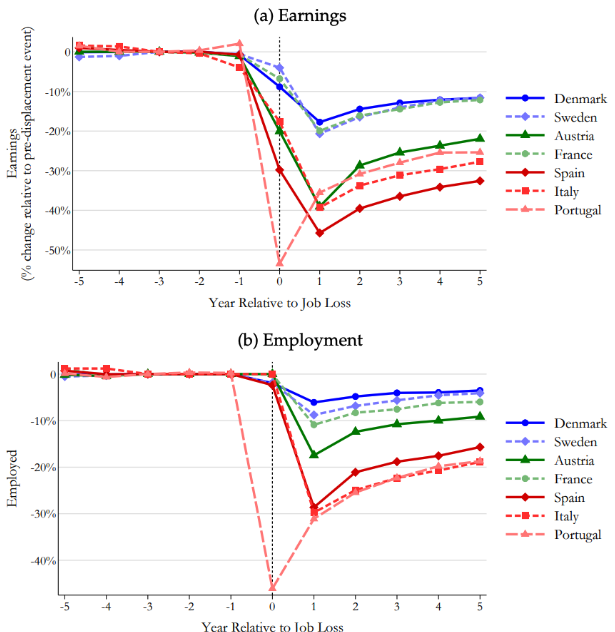
	Employed at Offer	Non-Employed at Offer	Difference, E-NE
Percent of job offers	72.1	27.9	
Offer Wage Estimates			
log real offer wage, unconditional	2.977 (0.031)	2.615 (0.047)	0.362 (0.101)
Controlling for worker & job characteristics	2.933 (0.026)	2.739 (0.031)	0.194 (0.048)
Additional Job Offer Characteristics			
log offer usual hours	3.396 (0.025)	3.269 (0.038)	0.126 (0.059)
Pct. of offers with no benefits	40.5 (1.7)	62.0 (3.0)	−21.5 (4.8)
Pct. of offers through an unsolicited contact	25.0 (1.5)	15.9 (2.3)	9.1 (3.5)
Pct. of offers with some counter-offer given	12.3 (1.2)	—	—
Pct. of offers that involved bargaining	38.0 (1.7)	25.8 (2.7)	12.2 (4.3)
Pct. of offers accepted as only option, conditional on acceptance	7.7 (1.6)	26.5 (3.9)	−18.8 (8.0)
Accepted Wage Estimates			
log real accepted wage, unconditional	3.000 (0.042)	2.542 (0.068)	0.458 (0.170)
Controlling for worker & job characteristics	2.906 (0.029)	2.710 (0.033)	0.195 (0.065)
Prior-Job Wage Estimates			
log real prior wage, unconditional	2.881 (0.041)	2.758 (0.053)	0.122 (0.088)
Controlling for worker & job characteristics	2.840 (0.036)	2.832 (0.044)	0.008 (0.071)

On-the-job search model

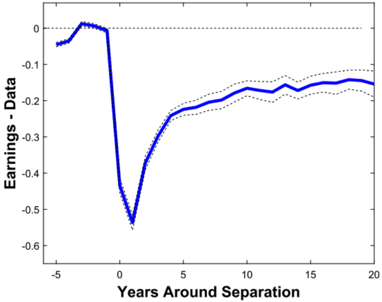
- The empirical results suggest that on-the-job search is pervasive, elastic, and dominates job search while unemployed along several margins
- Faberman et al. (2022) show that a classic models of on-the-job search with endogenous search effort and wage bargaining augmented with differences in search efficiency and bargaining power by employment status can account for the empirical findings
- In the model, unemployed are willing to accept low-paying job offers despite a relatively high flow value of unemployment because they are better off accepting a low-paying job so they can get on the job ladder and enjoy the efficiency of on-the-job search
- As employed workers climb the job ladder, they will reduce their search effort
- While much of the observed wage offer premium enjoyed by the employed reflects unobservable skills, a nontrivial fraction is explained by censoring and bargaining

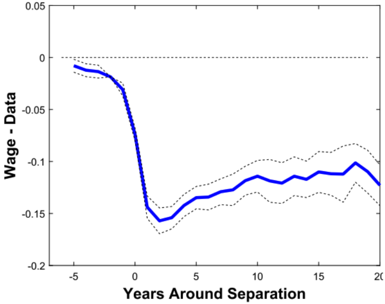
Recent literature on Job loss impact (due to mass layoff)

Figure 1: The Effect of Job Loss across Countries

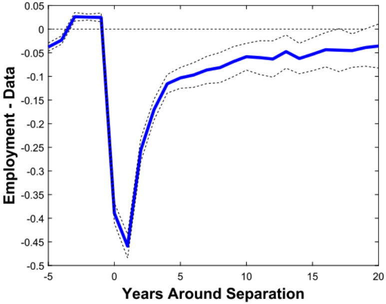


The job loss impact can persist fairly long (German)





(a) Wage Response



(b) Employment Response

Job loss and human capital depreciation

- The magnitude and persistence of these employment and earnings losses elude the workhorse models of job search
- Jarosch (2023) suggests that workers climb the job ladder towards both increasingly productive and more secure jobs so that unemployment spells are serially correlated
- Following the work of Ljungqvist and Sargent (1998), a common method for generating persistent earnings losses in quantitative models is to introduce human capital declines (depreciation) during unemployment (and human capital increases during employment)
- In Huckfeldt (2022), the declines in human capital during unemployment move a worker's human capital below the skill requirements of their prior occupation, resulting transition to a lower-paying occupation
- Braxton and Taska (2023) suggests that such occupation switching are concentrated among workers who are more exposed to technological changes that raises the skill requirements in prior jobs

Job loss and occupation switch

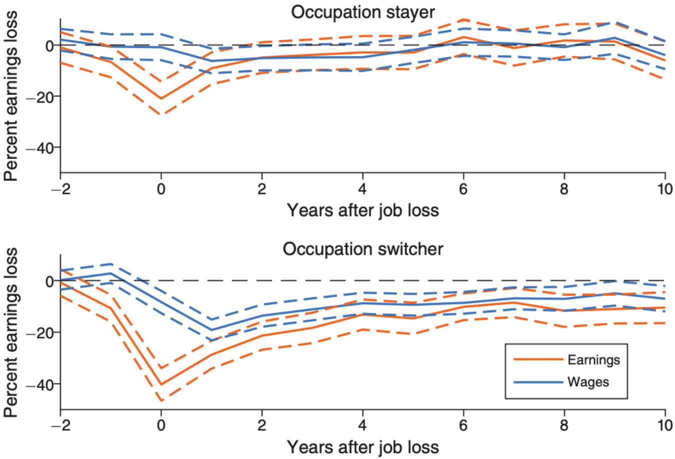


Figure 1. Earnings and Wage Losses Are More Persistent for Occupation Switchers

Notes: Estimates come from the PSID. Dashed lines represent 95 percent confidence intervals around estimates.

Job loss and technological change

Figure 2: Technological change and earnings losses after displacement

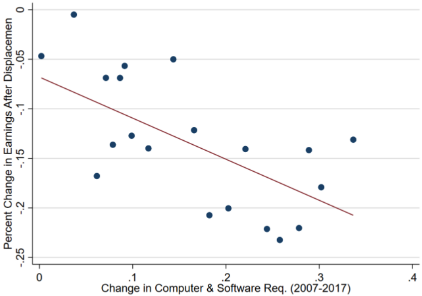
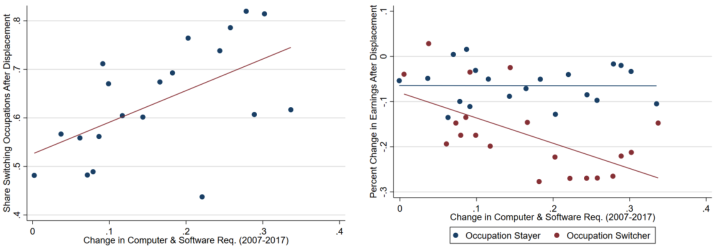


Figure 3: Technological change and occupation switching

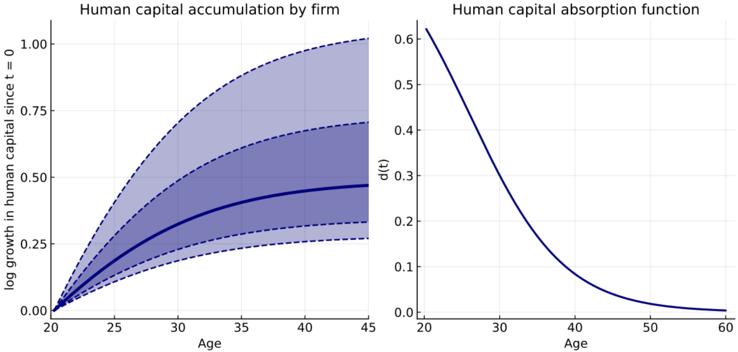
(a) Switching occupations

(b) Earnings losses by occupation switching



Firms as learning environments

- ▷ If human capital accumulation can be different between employment and unemployment, it is then natural to think that it can also vary across different firms
- ▷ Thus the learning environment attached with the job offer will also affect one's job search behavior, though its importance of course depends on the workers' age (recall human capital theory)
- ▷ Gregory (2020):



With search friction, luck begins to matter for your life, and HC accumulation magnifies its importance

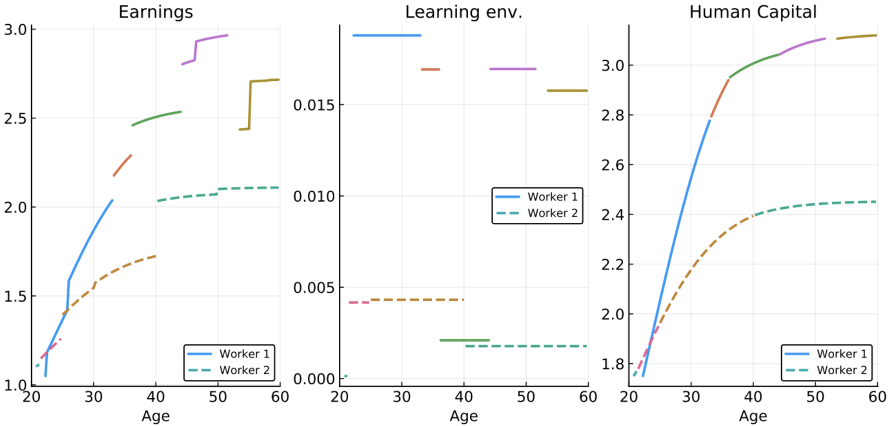


Figure 2: Example paths for workers with same learning ability.
The left panel shows earnings paths for two workers in the solid and dashed lines. Both have the same learning ability, but receive a different series of shocks over their lifetimes. Each separate color represents a spell in a different firm. Gaps (can be seen best in the human capital paths) represent unemployment spells. The middle panel shows the corresponding learning environments of the firms the workers match to. The right panel shows each worker's human capital profile.

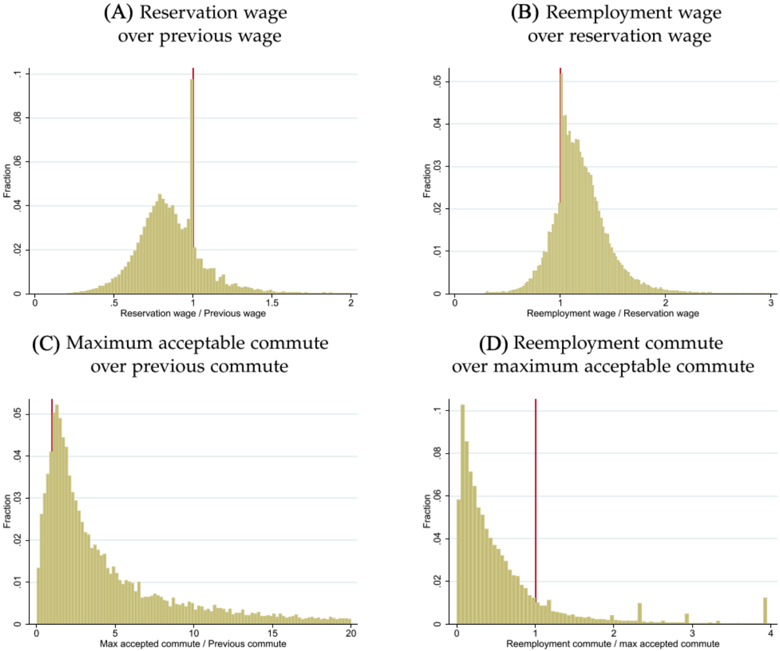
Job search and commuting time

- If the learning opportunity matters, it is then natural to consider that other job characteristics we considered in compensating differential would also matter for job search behavior

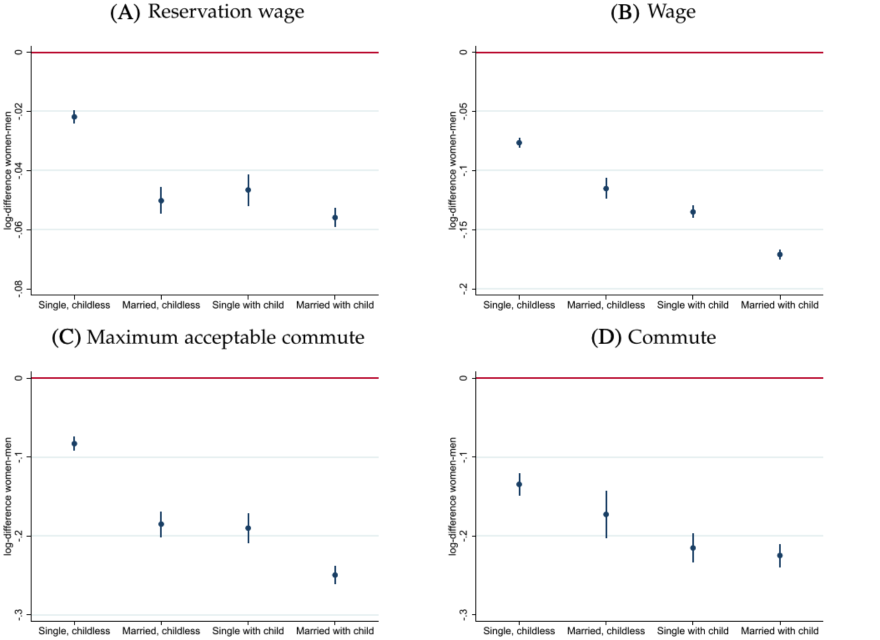
- Le Barbanchon et al. (2021) shows that workers care about commuting distance/time, especially for female workers
  - In OECD countries, women on average spend 22 minutes a day commuting, while men spend 33 minutes
  - They exploiting a unique feature of French institutions: registered job seekers must declare minimum wage and maximum commute
  - They find unemployed women have 4% lower reservation wage and 14% lower maximum acceptable commute
  - They suggest differences in commute valuation come from individual preferences or constraints resulting from household decisions

- A job search model extended with additional job attributes like commute would generate a reservation wage curve that gives for every commute the lowest wage that the job seeker is willing to accept

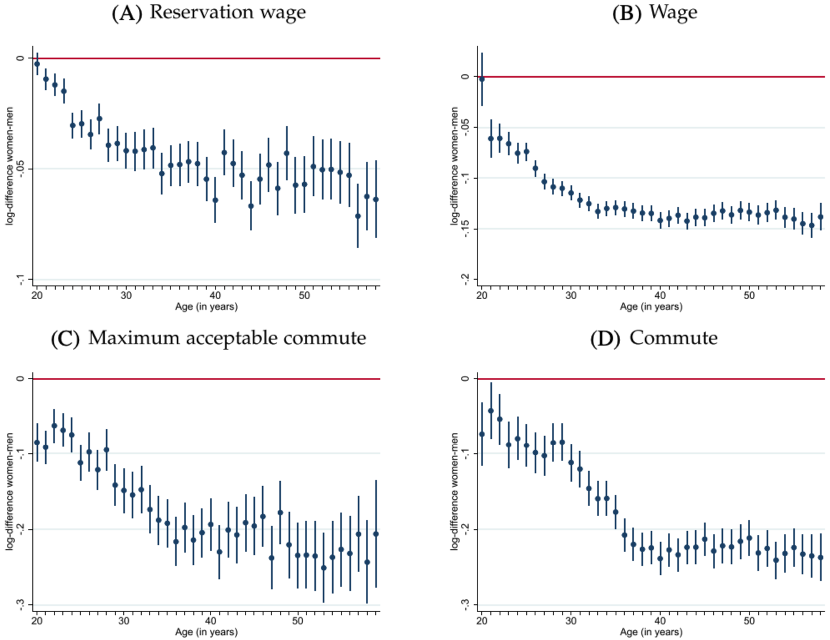
Job search and commuting



Job search gender gap by family type



Gender gap by age



Job search gender gap in job search

- A new classes of explanations on gender gap (e.g. educational or occupational or job choices, and earnings expectations) focus on gender differences in psychological attributes
  - Women exhibiting a greater degree of risk aversion
  - Men displaying a greater degree of overconfidence in their ability
  - They matter because we need to make decisions under uncertainty
- Cortés et al. (2023) apply these insights to job search behavior
  - They conduct both survey and labexperiment on undergraduates, who face considerable uncertainty in their first job and early-career
- They find
  - Women, on average, accept jobs about one month earlier than their male counterparts
  - A large gender gap in accepted offers, which narrows in favor of women over the course of the job search period
- They explain this in a search model:
  - If women have higher levels of risk aversion and less optimism regarding job offers, they will have lower reservation wages, start searching for jobs earlier, and accept jobs earlier

Gender gap in job search behavior

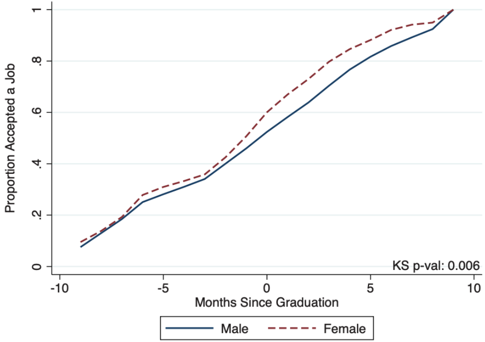
TABLE II

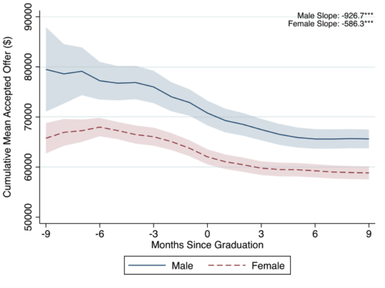
CONTINUED

	All	Men	Women	*p*-value
Search behavior (2018/2019 cohorts only)				
Observations	452	193	259	
Month start active job search	−3.96	−3.26	−4.49	.082
	(7.42)	(7.54)	(7.30)	
Total number of applications	75.22	94.67	60.72	.002
	(118.28)	(147.32)	(88.37)	
Offers per 100 applications	13.86	11.67	15.50	.088
	(23.48)	(22.71)	(23.95)	
Hours spent searching per week	9.61	10.30	9.10	.120
	(8.05)	(7.97)	(8.09)	
Proportion of jobs underqualified for	25.43	26.97	24.28	.124
	(18.40)	(18.17)	(18.52)	
Usefulness of career center in search (1–5)	2.41	2.19	2.57	.002
	(1.26)	(1.23)	(1.26)	

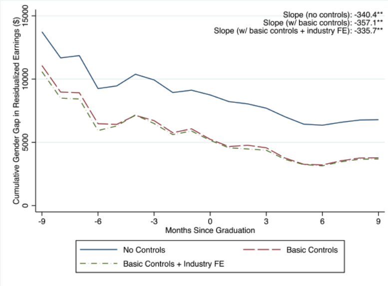
Notes. Variables in the Search behavior panel were collected in the postgraduation survey and refer to the entire job search period. The last column reports the *p*-value of the test of equality of means across gender. Earnings measures are expressed in 2017 dollars. "Accept job before grad" is a dummy variable indicating that the respondent had accepted a job offer before graduation. "Month accept offer" and "Month start active job search" are defined relative to the month of graduation (indicated as 0). "Time given to consider" is the deadline in weeks that the employer gave the respondent to accept or reject an offer. "Referral helped get job" is a dummy variable indicating that a referral helped the respondent get their first job. "Usefulness of career center in search" is based on the question of how useful the career center was in helping the respondent get

Gender gap in job search behavior





(A) Cumulative Mean Accepted Earnings



(B) Cumulative Gender Earnings Gap (M − F)

Search intermediaries

- As we have seen, rather than waiting for purely random offers coming out of thin air, workers more or less target their specific F based on their preference and attributes
- What could also play an important role is the various labor market intermediaries that connects job seekers with employers
- Job seekers use a variety of social connections (e.g. Dustmann et al. 2016; Gee et al. 2017)
  - former colleagues; friends and classmates from college; family ties
  - neighborhood-based social networks
  - individuals who belong to the same immigrant or ethnic community
  - Studies in general find connections and referrals help individuals obtain employment and higher earnings
- Online job boards, or more generally, internet, have become the major channel for worker-firm matching in recent days
  - While it's straightforward to think internet reduces cost of searching and posting, evidences of its effectiveness only from a few studies focusing on early internet age (e.g. Kuhn and Mansour 2014), and there are potential counteracting effects (Martellini and Menzio, 2020)

Comparing different search channels

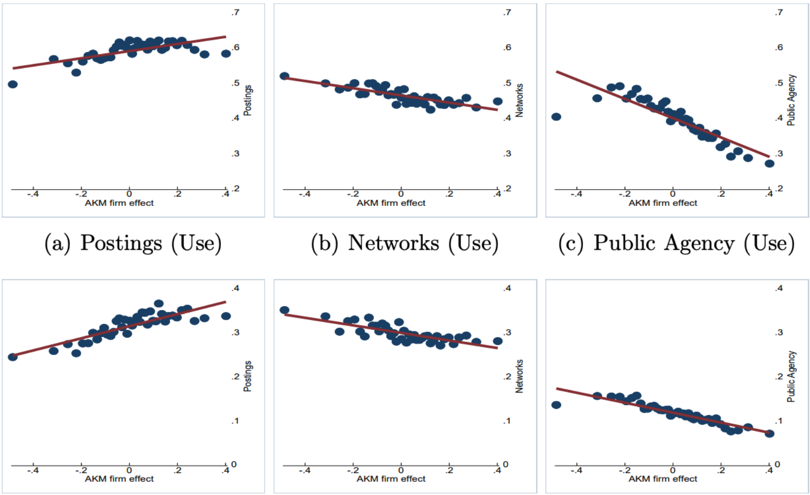
- Carrillo-Tudela et al. (2023) studies different channels that firms and workers can use to match with each other and their impact on matching results

Table 1: Use and success of search channels

	Firms (JVS)		Workers (PASS)	
Search channel	Use (%)	Successful (%)	Use (%)	Successful (%)
Postings	55.3	28.7	88.1	18.7
Networks	54.1	40.5	60.22	27.0
Public Agency	37.7	13.3	57.3	8.4
Unsolicited	18.7	8.0	-	-
Internal	14.5	5.3	-	-
Private Agent	6.1	2.6	12.1	2.2
Others	2.7	1.5	16.9	43.7
Total	**189.0**	**100.0**	**234.6**	**100.0**

- They find that high-wage firms and high-wage workers job posting use and succeed more through job posting

Search channel of firms

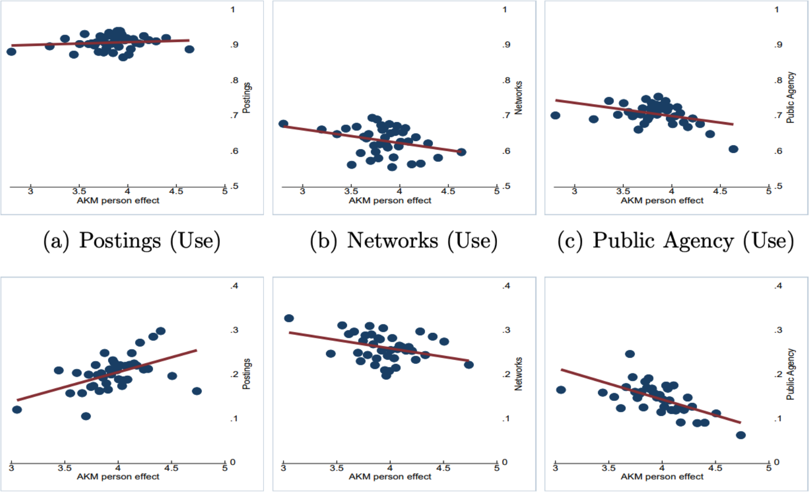


(a) Postings (Use) (b) Networks (Use) (c) Public Agency (Use)

(d) Postings (Success) (e) Networks (Success) (f) Public Agency (Success)

Figure 1: Use and success of search channels by AKM firm fixed effect

Search channel of workers

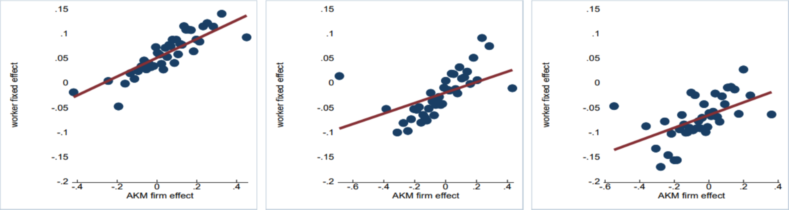


(a) Postings (Use) (b) Networks (Use) (c) Public Agency (Use)

(d) Postings (Success) (e) Networks (Success) (f) Public Agency (Success)

Figure 2: Use and success of search channels by AKM worker fixed effect

Search channel and worker-firm sorting



(a) Postings (b) Networks (c) Public Agency

Figure 4: Relationship between worker and firm AKM fixed effect by hiring channel

Notes: The figures show binscatter plots that relate the firm AKM fixed effect to the AKM fixed effect of the worker hired by this firm separately for each of the three channels "Postings", "Networks", or "Public Agency". The same controls as in Figure 1 are applied.

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Appendix

Sequential job search model: formal notation

- Specify the class of decision rules of the agent:
  - Action: $a_t : \mathbb{W} \to [0,1]$ specifies the agent's acceptance probability for each wage in $\mathbb{W}$ at time t; Let $a_t' \in \{0,1\}$ be the realization
  - Let A_t denote the set of realized actions by the individual, and define $A^t = \prod_{s=0}^{t} A_s$
  - Then a strategy for the individual $p_t : A^{t-1} \times \mathbb{W} \to [0,1]$
  - Let $\mathcal{P}$ be the set of such functions (with the property that $p_t(\cdot)$ is defined only if $p_s(\cdot) = 0$ for all $s \leq t$) and $\mathcal{P}^\infty$ the set of infinite sequences of such functions

- Most general way of expressing the problem of the individual: $\max_{\{p_t\}_{t=0}^{\infty} \in \mathcal{P}^\infty} \mathbb{E} \sum_{t=0}^{\infty} \beta^t c_t$ s.t. $c_t = b$ if $t < s$ and $c_t = w_s$ if $t \geq s$ where $s = \inf \{n \in \mathbb{N} : a_n' = 1\}$

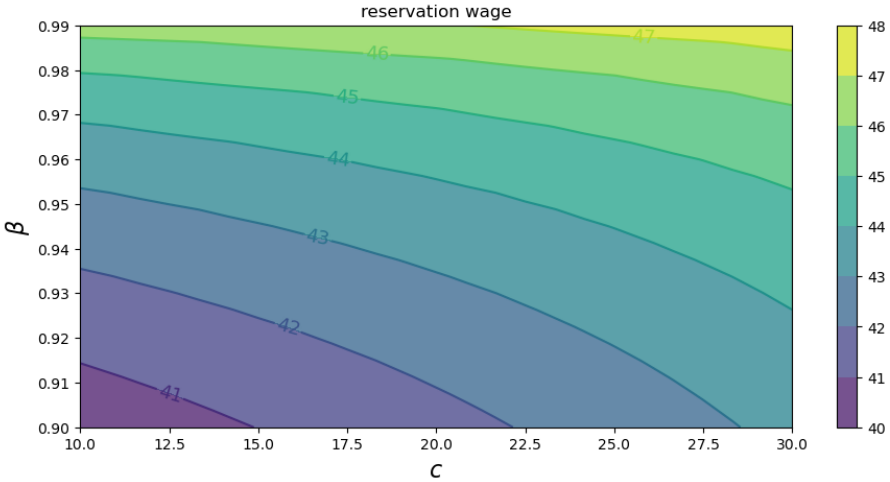
Reservation wage expressions

- $\frac{R}{1-\beta} = b + \beta \int V(w)dF(w) = b + \beta \left[\frac{RF(R)}{1-\beta} + \int_{w \geq R} \frac{w}{1-\beta} dF(w)\right]$
- One way: $R = \frac{1}{1-\beta F(R)} \left[b(1-\beta) + \beta \int_R^\infty w dF(w)\right]$
- More useful: $\int_{w<R} \frac{R}{1-\beta} dF(w) + \int_{w \geq R} \frac{R}{1-\beta} dF(w) =$
  $b + \beta \left[\int_{w<R} \frac{R}{1-\beta} dF(w) + \int_{w \geq R} \frac{w}{1-\beta} dF(w)\right]$
- Minus $\beta \int_{w<R} \frac{R}{1-\beta} dF(w) + \beta \int_{w \geq R} \frac{R}{1-\beta} dF(w)$ in both side, we get
  $R = b + \beta \left[\int_{w \geq R} \frac{w-R}{1-\beta} dF(w)\right]$
- $\Rightarrow R - b = \frac{\beta}{1-\beta} \left[\int_{w \geq R} (w-R) dF(w)\right] \equiv g(R)$
  - Intuition: cost of foregoing R = expected benefit of one more search
  - $g'(R) = -\frac{\beta}{1-\beta}[1 - F(R)] < 0$ implies a unique solution since LHS increases in R and RHS decreases in R
- Use integration by parts, we can also write this as
  $R = b + \frac{\beta}{1-\beta} \int_{w \geq R} [1 - F(w)]\, dw$

Solving the reservation wage R

- In the basic model, we have $R - b = \frac{\beta}{1-\beta}\left[\int_{w \geq R}(w - R)dF(w)\right]$
- Simultaneously add and minus $\frac{\beta}{1-\beta}\left[\int_{w \leq R}(w - R)dF(w)\right]$ on the RHS, we have
  $R - b = \frac{\beta}{1-\beta}(Ew - R) - \frac{\beta}{1-\beta}\left[\int_{w \leq R}(w - R)dF(w)\right]$, where
  $Ew = \int wdF(w)$ = mean of the wage distribution
- $\Rightarrow R - b = \beta(Ew - b) - \beta\int_{w \leq R}(w - R)dF(w)$
- $\Rightarrow R - b = \beta(Ew - b) + \beta\int_{w \leq R}F(w)dw$ (use integration by parts)
  - Although we still have R in both RHS and LHS, they move in an unambiguous way, and thus we are able to do comparative statics

Reservation wage solved numerically



Aside on Riskiness and Mean Preserving Spreads

- Definition. A distribution $F(x, r)$ over X is less risky than $F(x, r')$, written as $F(x, r) \succeq_R F(x, r')$, if for all concave and increasing $u : \mathbb{R} \to \mathbb{R}$, we have $\int_X u(x)dF(x, r) \geq \int_X u(x)dF(x, r')$
- At some level, it may be a more intuitive definition of "riskiness" to require that $F(x, r)$ and $F(x, r')$ to have the same mean, i.e., $\int_X xdF(x, r) = \int_X xdF(x, r')$ while still $F(x, r) \succeq_R F(x, r')$
- Definition. $F(x, r)$ second order stochastically dominates $F(x, r')$, written as $F(x, r) \succeq_{SD} F(x, r')$, if $\int_{-\infty}^{c} F(x, r)dx \leq \int_{-\infty}^{c} F(x, r')dx$, for all $c \in X$
- Theorem. (Blackwell, Rothschild and Stiglitz) $F(x, r) \succeq_R F(x, r')$ if and only if $F(x, r) \succeq_{SD} F(x, r')$
- Thus mean preserving spreads are essentially equivalent to second-order stochastic dominance with the additional restriction that both distributions have the same mean

Continuous-time model

- First, generalize the discrete-time model to allow the length of a period to be Δ: $\beta = \frac{1}{1+r\Delta}$ and assume worker gets a wage offer with probability $\alpha\Delta$ in each period ‣ often introduced as a Poisson arrival rate

- The values now become: $W(w) = \Delta w + \frac{1}{1+r\Delta} W(w)$,
  $U = \Delta b + \frac{\alpha\Delta}{1+r\Delta} \times \int \max\{U, W(w)\} dF(w) + \frac{1-\alpha\Delta}{1+r\Delta} U$

- $\Rightarrow rW(w) = (1 + r\Delta)w$,
  $rU = (1 + r\Delta)b + \alpha \int \max\{0, W(w) - U\} dF(w)$

- When $\Delta \to 0$, we obtain the continuous time Bellman equations:
  $rW(w) = w$, $rU = b + \alpha \int \max\{0, W(w) - U\} dF(w)$
  - Intuitively, the flow rate of value functions equals the sum of the instantaneous payoff, plus the expected value of any changes in the value of the worker's state

- Reservation wage satisfies $W(R) = U$, which implies
  $W(w) - U = (w - R)/r$ and $R = b + \frac{\alpha}{r} \int_{w>R} (w - R)\, dF(w)$

Continuous-time model: search intensity

- It's natural to consider endogenize search intensity (effort) under the continuous-time model
- Suppose a worker can affect the arrival rate of offers α, at cost $g(\alpha)$, where $g' > 0$ and $g'' > 0$
- Unemployed workers choose α to maximize $rU = R$, where $R = b - g(\alpha) + \frac{\alpha}{r} \int_{w>R} (w - R)\, dF(w)$
- The FOC for an interior solution is $\int_{w>R} (w - R)\, dF(w) = rg'(\alpha)$
- Worker behavior is characterized by a pair (R, α) solving the system; It easy to show that an increase in b raises R and reduces α
- The probability that the worker has not found a job after a spell of length t is e^{-Ht}, where $H = \alpha [1 - F(R)]$ is the hazard rate; The average duration of an unemployment spell is $D = \int tHe^{-Ht} dt = \frac{1}{H}$

Continuous-time model: unemployment

- Assume jobs end for some exogenous reason (e.g. layoff risk) and this occurs according to a Poisson process with parameter λ
- U does not change; $rW(w) = w + \lambda[U - W(w)]$; Reservation wage: $R = b + \frac{\alpha}{r+\lambda} \int_{w>R} [w - R] dw$
- Note a worker now goes through repeated spells of employment and unemployment: when unemployed, he gets a job at rate $H = \alpha [1 - F(R)]$, and while employed he loses the job at rate λ
- A simple way to endogenize transitions to unemployment is to allow w to change at a given job and worker to quit the job
  - A Poisson process with λ that a new w' is drawn from $F(w' \mid w)$
  - The exogenous layoff model discussed above is a special case where $w' = 0$ with probability 1
- $rW(w) = w + \lambda \int \max\{W(w') - W(w), U - W(w)\} dF(w' \mid w)$
  - In the simplest case where $F(w' \mid w) = F(w)$ (independence), we have $R = b + \frac{\alpha - \lambda}{r+\lambda} \int_{w_R}^{\infty} (w - w_R) dF(w)$ (Q: when $R < b$?)

Continuous-time model: on-the-job search

- Suppose new offers arrive at rate α_0 while unemployed and α_1 while employed; Each offer is an i.i.d. draw from F
- Assume employed workers lose their job exogenously at rate λ
- Bellman equations: $rU = b + \alpha_0 \int_{x>R} [W(w) - U] dF(w)$; $rW(w) = w + \alpha_1 \int \max\{W(w') - W(w), 0\} dF(w') + \lambda [U - W(w)]$
- As W is increasing in w, unemployed workers use a reservation wage satisfying $W(R) = U$, and employed workers switch jobs whenever $w' > w$
- Combine two equations at $w = R$ we have:
  $R = b + (\alpha_0 - \alpha_1) \int_{w>R} [W(w') - W(R)] dF(w')$
  - Note $R < b$ if $\alpha_0 < \alpha_1$: if a worker gets offers more frequently when employed, he is willing to accept wages below b
- $\Rightarrow R = b + (\alpha_0 - \alpha_1) \int_{w>R} \left[\frac{1-F(w)}{r+\lambda+\alpha_1[1-F(w)]} \right] dw$ (use $W'(w)$)
  - As before $\partial R / \partial b > 0$, and thus an increase in UI reduces turnover

Continuous-time model: aggregation

- In a case where many workers each solving a problem and various stochastic events are i.i.d., we can also discuss aggregate variables
- Each unemployed worker becomes employed at rate $H = \alpha_0 \left[1 - F\left(R\right)\right]$; Each employed worker loses his job at rate λ
- Thus aggregate unemployment rate evolves as $\dot{u} = \lambda(1-u) - \alpha_0 \left[1 - F\left(R\right)\right] u$
- Over time, this converges to the steady state: $u^* = \frac{\lambda}{\lambda + \alpha_0 [1 - F(R)]}$
- For any wage $w \geq R$,
  - The flow of workers into employment at a wage no greater than w is $u\alpha_0 \left[F(w) - F\left(w_R\right)\right]$;
  - The flow out is $(1-u)G(w)\left\{\lambda + \alpha_1[1 - F(w)]\right\}$, where $G(w)$ is the cdf of the observed wage distribution
  - Combining two in steady state: $G(w) = \frac{\lambda[F(w) - F(w_R)]}{[1 - F(w_R)]\{\lambda + \alpha_1[1 - F(w)]\}}$
- The steady state job-to-job transition rate: $\alpha_1 \int_{w_R}^{\infty} [1 - F(w)] dG(w)$

Poisson process

- Alternatively, we may assume that the number of wage offers per period is a Poisson random variable with mean $\alpha\Delta$
- In fact, in dynamic economics models of all fields, an event is often described as a "Poisson process" or having a "Poisson arrival rate"
- What is Poisson process? A Poisson process $\{N(t), t \geq 0\}$ with rate $\lambda > 0$ can be defined as a counting process satisfying
  - (i) Initialization: $N(0) = 0$
  - (ii) Independent Increments: For any $0 \leq t_1 < t_2 < \ldots < t_n$, the random variables $N(t_2) - N(t_1), \ldots, N(t_n) - N(t_{n-1})$ are independent
  - (iii) Stationary Increments: For any $s, t \geq 0$, $N(t+s) - N(s)$ has the same distribution as $N(t)$.
  - (iv) Poisson Distribution of Increments: For any $t \geq 0$, $N(t)$ has a Poisson distribution with parameter λt, i.e., $P(N(t) = k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}$ for $k = 0, 1, 2, \ldots$
  - (v) No Simultaneous Events: $P(N(t) > 1) = o(t)$ as $t \to 0$
- The parameter λt is also the mean of the corresponding Poisson distribution, and hence the mean of offer # per period t

Poisson process and exponential variable

- For a Poisson process, the waiting times between successive meetings are i.i.d. exponential variables with mean $1/\lambda$
- The i.i.d property directly comes from the independent/memoryless nature of Poisson process
- To see "exponential", consider the probability that the first event (or the next event) occurs after time t, i.e. no events occurring in the interval $[0, t]$: $P(N(t) = 0) = \frac{e^{-\lambda t}(\lambda t)^0}{0!} = e^{-\lambda t}$
- Thus the probability that the waiting time T is less than t, i.e. at least one event occurring, is $P(T \leq t) = P(N(t) > 0) = 1 - e^{-\lambda t}$
- This is exactly the CDF of the exponential distribution, with pdf $f(t) = \lambda e^{-\lambda t}$ and mean $1/\lambda$

Poisson process and arriving rate

- Recall that from the the Poisson distribution we have $P(N(t)=0) = \frac{e^{-\lambda t}(\lambda t)^0}{0!} = e^{-\lambda t} \approx 1 - \lambda t + o(t)$
  - The last approximation uses the Taylor series expansion of an exponential function e^x around 0: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$
  - $o(t)$ captures higher-order terms in the Taylor expansion that become negligible as t approaches zero
  - In formal terms, $f(t)$ is $o(t)$ if the limit of $f(t)/t$ as $t \to 0$ is 0
- Similarly, the probability of exactly one event occurring in a small time interval t: $P(N(t)=1) = \frac{e^{-\lambda t}(\lambda t)^1}{1!} \approx \frac{(1-\lambda t)(\lambda t)}{1} = \lambda t - \lambda^2 t^2$
  - For very small t, $\lambda^2 t^2$ becomes negligible compared to λh, so we can also write $P(N(t)=1) \approx \lambda t + o(t)$
- Finally, consider a very small time interval/unit $t = 1$, all $o(t)$ terms approach to 0, and we have the arriving rate $P(N(t)=1) \approx \lambda$ and the non-occurrence rate $P(N(t)=0) \approx 1 - \lambda$
  - Note $\lambda + 1 - \lambda = 1$ as the probability of more than one event occurring $P(N(t) > 1)$ also becomes negligible and vanishes