

Wage Structure: Compensating Differential

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Fall, 2023

Roadmap

1. Introduction

2. Compensating differentials: basics

3. Compensating differentials: applications

Introduction

- ▷ In most models of labor market we have learned, the only thing that matters for workers to choose across different jobs and firms are the pay (Q: which is the exception?)
- ▷ If this is true, everyone should have the same firm ranks (even with idiosyncratic preference on money/consumption)
- ▷ When you graduate and look for a job, how do you choose which companies to apply to? (Q: why did you choose Keio but not other schools? why did you choose Japan but not other countries?)

Top popular firms among new graduates in Japan

順位	企業/団体	昨年比順位	順位	企業/団体	昨年比較順位
1	三菱商事	1(→)	26	東京ガス	55(↑)
2	伊藤忠商事	3(↑)	26	三菱地所	9(↓)
3	三井物産	2(↓)	28	博報堂/博報堂DYメディアパートナーズ	18(↓)
4	住友商事	4(→)	29	三井不動産	15(↓)
5	丸紅	7(↑)	29	NTTドコモ	34(↑)
6	野村総合研究所	5(↓)	31	富士フイルム	30(↓)
7	三井住友銀行	6(↓)	32	マッキンゼー・アンド・カンパニー(McK)	74(↑)
8	三菱UFJ銀行	8(→)	33	Google	49(↑)
9	サントリーホールディングス	11(↑)	34	アビームコンサルティング	49(↑)
10	アクセンチュア	19(↑)	35	三井住友信託銀行	23(↓)
11	NTTデータ	13(↑)	35	トヨタ自動車	42(↑)
12	ソニーグループ	17(↑)	37	日本航空(JAL)	90(↑)
13	楽天グループ	12(↓)	38	関西電力	32(↓)
14	みずほフィナンシャルグループ	27(↑)	38	東京海上日動火災保険	20(↓)
14	PwCコンサルティング	29(↑)	40	電通	24(↓)
16	旭化成	10(↓)	40	日本郵船	22(↓)
17	JR東海	14(↓)	40	AGC	59(↑)
18	双日	25(↑)	43	野村證券	67(↑)
19	農林中央金庫	31(↑)	44	大阪ガス	119(↑)
20	日本政策投資銀行	16(↓)	45	三菱UFJ信託銀行	38(↓)
21	味の素	21(→)	45	全日本空輸(ANA)	119(↑)
22	ボストン コンサルティング グループ(BCG)	73(↑)	47	KDDI	37(↓)
23	豊田通商	57(↑)	48	花王	47(↓)
24	デロイトトーマツ コンサルティング	49(↑)	48	日本IBM	81(↑)
25	住友化学	25(→)	48	富士通	39(↓)

Top factors of firm attractiveness viewed by Japanese graduates

24年卒春

トップ10

- 1 将来的な高所得の見通し(報酬と昇進の機会)
future high income
- 2 企業の強い社会的責任感(企業の評判とイメージ)
social responsibility
- 3 チャレンジ性のある仕事(仕事の特性)
challenging job
- 4 将来のキャリアへの良い関連性(報酬と昇進の機会)
future career
- 5 友好的な職場環境を提供する(人と文化)
friend work environment
- 6 魅力的または面白い製品サービス(企業の評判とイメージ)
attractive products/service
- 7 多様な仕事内容(仕事の特性)
multi-faceted job content
- 8 仕事と生活の良いバランス(人と文化)
work-life balance
- 9 競争力のある福利厚生(報酬と昇進の機会)
competitive welfare
- 10 市場での成功(企業の評判とイメージ)

23年卒春

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- 10 成長をサポートするリーダーがいる(人と文化)

Japanese Black company

≡ Black company (Japan)

🌐 7 languages ▾

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From Wikipedia, the free encyclopedia

This article is about the Japanese term. For other uses, see [Black company](#).

A **black company** (ブラック企業, *burakku kigyō*), also referred to in English as a **black corporation** or **black business**, is a Japanese term for an [exploitative](#), [sweatshop](#)-type employment system.

While the term "sweatshop" is associated with [manufacturing](#), and the garment trade in particular, in [Japan](#) black companies are not necessarily associated with the [clothing industry](#), but more often with office work.

Etymology [\[edit \]](#)

The term "black company" was coined in the early 2000s by young IT workers but has since come to be applied to various industries.^[1]

Conditions [\[edit \]](#)

While specifics may vary from workplace to workplace and company to company, a typical practice at a black company is to hire a large number of young employees and then force them to work large amounts of [overtime](#) without overtime pay. Conditions are poor, and workers are subjected to verbal abuse and "power harassment" (bullying) by their superiors.^[1] In order to make the employees stay, superiors of black companies would often threaten young employees with disrepute if they chose to quit.

Noteworthy cases [\[edit \]](#)

Mina Mori, a 26-year-old employee of the restaurant chain [Watami](#), committed suicide^[2] two months after joining the company in 2008. Her family lodged a complaint with the [Yokosuka](#) Labor Standards Office to seek recognition of the suicide as work-related. When their claim was denied, they appealed it to the [Kanagawa](#) Prefectural [Labor Bureau](#), which recognized work-related stress as the cause of the decline of her mental health.^{[3][4]} In December 2017, the company was fined 1.1 million yen for failing to provide adequate support for Mori's mental health.

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2. Compensating differentials: basics

3. Compensating differentials: applications

Theory of compensating differentials (Rosen, 1986)

- ▷ Heterogenous firm provide jobs with various attributes (amenities or disamenities) in the labor market
 - ▷ These attributes can be either pecuniary or nonpecuniary: composition of pay packages; working conditions; work-time scheduling; fringe benefits; job secularity; career potential; on-the-job training/learning; location and commuting time; ...
 - ▷ Different firms have different cost functions in providing amenities
- ▷ Heterogenous workers have idiosyncratic preferences/tastes on each job attributes
 - ▷ All attributes of jobs are common knowledge and observed by all workers and firms
 - ▷ The idiosyncratic preference is asymmetric information that is only known by the worker him/herself
- ▷ Market equilibrium is competitive, i.e. both firms and workers take equilibrium market prices as given ($\# \text{ firms} \gg \# \text{ of job types}$)
 - ▷ Labor markets for each job type must clear (labor demand = labor supply)
 - ▷ The equilibrium also features a matching & sorting function of assigning specific workers to specific firms

Workers' job choices

- ▷ Assume two job types in the market: $D = 1, 0$ (i.e. with disamenity or not, e.g. dirty environment or distant workplace)
- ▷ Assume workers are productively homogeneous (relax next week) but have heterogenous utility function: $u_i = u_i(C, D)$
 - ▷ $u_i(C, 0) \geq u_i(C, 1)$
 - ▷ $u_i(C^*, 1) = u_i(C_0, 0)$ with $C^* > C_0$
 - ▷ Define $Z_i = C^* - C_0$ as the compensating variation
- ▷ Competitive labor markets implies two offers $(w_0, 0)$ and $(w_1, 1)$ that workers take as given
 - ▷ Define $\Delta W = (w_1 - w_0)$ as the market equalizing difference
- ▷ A worker i chooses job type that maximizes utility: choose $D_i = 1$ or $D_i = 0$ as $\Delta W \gtrless Z_i$

Indifference curve under continuous job types

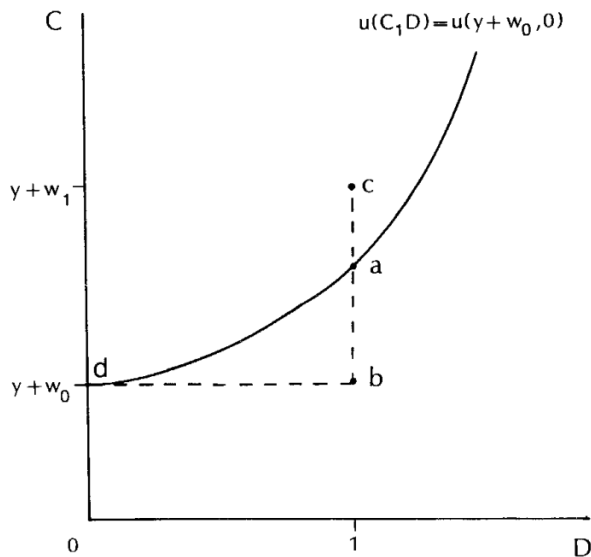


Figure 12.1

Market labor supply

- ▷ Given the size of the labor force, labor supply to each job type (market) are completely characterized by calculating the number of workers for whom $\Delta W \geq Z_i$
- ▷ Assume Z in the population has the probability density function (pdf) $g(Z)$, then the labor supply in the two markets given the market wages would be: $N_1^s = \int_0^{\Delta W} g(z) dz = G(\Delta W)$,
 $N_0^s = \int_{\Delta W}^{\infty} g(z) dz = 1 - G(\Delta W)$
- ▷ (Relative) Wage elasticity of labor supply is decreasing in the variance or spread of the distribution $g(Z)$

Relative supplies of worker partition distribution of preference $f(Z)$

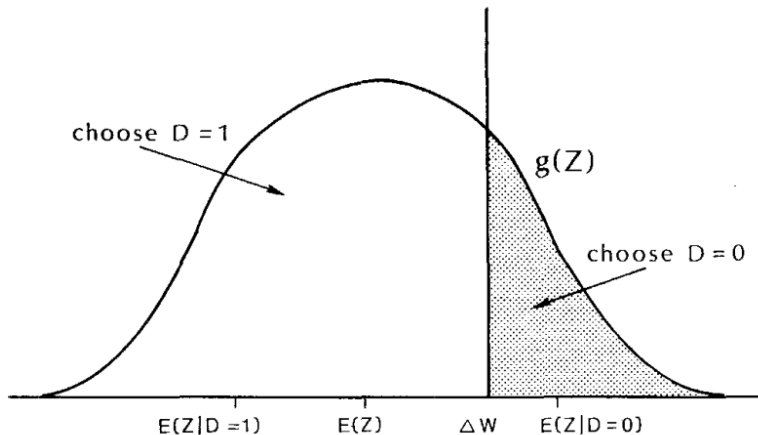


Figure 12.2

Firms' job choices

- ▷ A firm can spend resources to clean up its work environment, and the cost of doing so is compared with the wage savings on labor costs
- ▷ Assume each firm has a specific linear technology:
 $y_j = a_{j1}L$, if $D = 1$,
 $y_j = a_{j0}L$, if $D = 0$.
- ▷ Define $B_j = (a_{j1} - a_{j0})$, with restriction $B_j > 0$ i.e. $D = 1$ must be more productive (Q: what if $B_j < 0 \forall j$?)
- ▷ A worker j chooses job type that maximizes the profit: choose $D_j = 1$ or $D_j = 0$ as $B_j \gtrless \Delta W$ (Q: write the firm problem)

Market labor demand

- ▷ Assume exogenous firm size and normalize it to $L = 1$
- ▷ Assume B in the economy has the pdf $f(B)$, thus the labor demand in the two labor markets would be

$$N_1^d = \int_{\Delta W}^{\infty} f(B) dB = 1 - F(\Delta W), \quad N_0^d = \int_0^{\Delta W} f(B) dB = F(\Delta W)$$

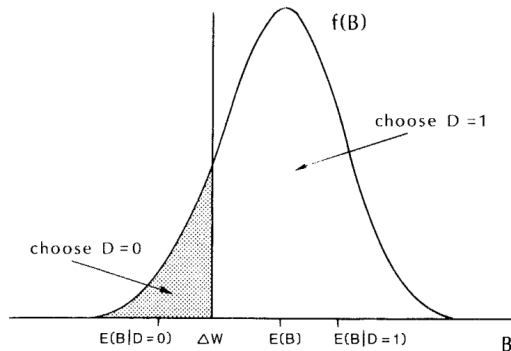


Figure 12.3

Market equilibrium

- ▷ In equilibrium, we have the market wages w_1 and w_0 such that two labor markets clear: $N_1^s = N_1^d$, $N_0^s = N_0^d$
- ▷ Workers and firms are systematically matched or assigned to each other in market equilibrium (i.e. "assortative matching"): workers with larger than average values of Z are found in firms with smaller than average values of B and conversely (Q: try rephrase this intuitively)
 - ▷ More formally $E(Z | D = 0) \geq E(Z)$ and $E(B | D = 0) \leq E(B)$ (i.e. selectivity bias), where $E(Z) = \int Zg(Z)dZ$ and $E(B) = \int Bf(B)dB$
- ▷ Marginal agents vs Inframarginal agents
 - ▷ Marginal workers and firms are indifferent from two choices
 - ▷ Inframarginal workers and firms earn positive economic rents and have 0 elasticity with a marginal change in the relative wage
 - ▷ Average economic rents earned by workers and firms are $\Delta W - E(Z | D = 1)$ and $E(Z | D = 0) - \Delta W$
 - ▷ see an example of detailed derivation under normal distribution

Generalize to more job types

- ▶ Let D take on K possible values, with $K \geq 2$
- ▶ Now k distinct markets must be considered, one for each value of D , competitive wage in the k th market is W_k , and a worker or a firm considers the choice set (W_k, D_k) , for $k = 1, 2, \dots, K$
- ▶ Nonetheless the ordering of optimal assignments by tastes and relative costs are more or less preserved
 - ▶ The underlying taste and technology distributions are partitioned into at most k ordered regions
 - ▶ Workers with the largest Z_i tend to be assigned to the jobs with smallest D_j offered by firms with smallest B_j , i.e. assortative matching feature of market equilibrium is generalized

Generalize to continuous job types

- ▷ A marginal analysis well serves to illustrate these ideas when D as a continuous variate, i.e. $D \in \mathbb{R}$
- ▷ The market price now becomes a wage function/profile: $W(D)$, which can be also regarded as an "equalizing difference function" (nondecreasing if D is a disamenity)
- ▷ Worker chooses D to maximize utility $u = U(W(D), D)$
- ▷ FOC: $-U_D = W'(D)U_C$
 - ▷ U_D/U_C is the MRS between D and consumption goods
 - ▷ But gradient of $W(D)$ is the correct (marginal) price in the optimization calculation, not $W(D)$ itself
 - ▷ The slop of the wage function, $W'(D)$, equals to the MRS, which is the slope of the indifference curve (see why), for the worker who happen to choose that particular value of D in the equilibrium

Equilibrium wage function $W(D)$

θ are worker indifference curves; ϕ are firm indifference curves

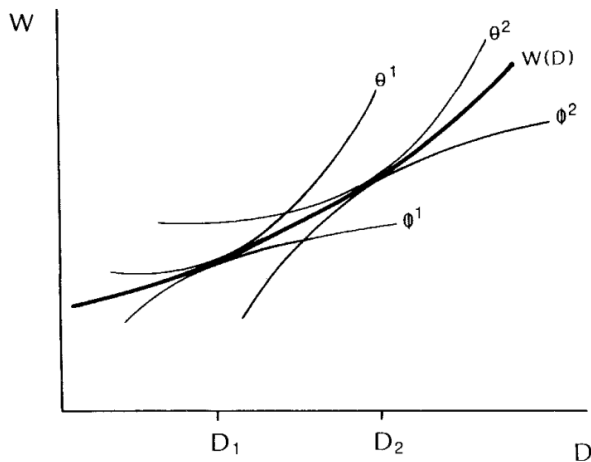


Figure 12.4

(Q: what much W required for worker 1 to move to D_2 ? And 2 to D_1 ?)

(Q: what would the figure be if either workers or firms are identical?)

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Work conditions

- ▶ Interesting examples mentioned in [Rosen \(1986\)](#)
 - ▶ Recruitment of labor for the Alaskan Pipeline, where the extreme severity of working conditions called for large wage premiums
 - ▶ The military uses elaborate pricing schemes to recruit personnel to various positions (e.g. logistics personnel vs. line soldiers)
 - ▶ Substantial differences in earnings between Lawyers specializing in public interest litigation vs. in more traditional practice
- ▶ More potential examples
 - ▶ Civil servant
 - ▶ Academic scholars
 - ▶ Animator illustrators in Japan
 - ▶ Part-time night job at convenience store
 - ▶ ...

Human capital investment

- ▷ Two types of post-school human capital investment considered in the literature
 - ▷ On-the-job training: often workers take the cost as foregone earnings
 - ▷ Learning-by-doing: learning is a by product of work, so no earnings are forgone; appear to be "a free lunch"
- ▷ Rosen (1972) applies equalizing difference framework to LBD and suggests that when a market for jobs with different learning content is introduced, a cost of acquiring skills is added and the two models become much more similar
 - ▷ Firms sell(supply) a job as a tied package of work and learning; Some jobs provide more learning opportunities and some provide less
 - ▷ Young workers (who have the largest incentive to learn under HC theory) are assigned to those jobs for which learning potential is largest but wages are depressed;
 - ▷ They subsequently move or are promoted across jobs that offer successively smaller learning opportunities but higher wages
 - ▷ Firms accommodate this by reallocating experienced workers' time away from direct production and toward instructing inexperienced

Human capital investment (cont.)

- ▷ If persons vary in the ability of learning [▶ see a simple model](#)
 - ▷ Workers with higher learning ability are assigned to jobs with greater learning opportunities at each age
 - ▷ The firms that have lower costs (higher capability) to provide greater learning chances are also likely to be those with high ability workers
- ▷ (Gregory (2020) shows that luck can also play an important role in life-cycle earning inequality when search friction exists)
- ▷ Real world examples:
 - ▷ Differences in earnings among medical specialists are largely equalizing on differential training costs and forgone earnings (e.g. see the [study by Friedman & Kuznets in 1954](#))
 - ▷ State legislators, judges, and many officials give up substantial alternative pay but are compensated by future private monetary rewards through the political capital built up
 - ▷ ...

Working hour

- ▷ In the labor supply analysis, worker is allowed to seek as much leisure as desired at a competitively determined wage rate
- ▷ However a firm may offer a fixed wage-hours package to workers, take it or leave it, with these package deals varying across firms
 - ▷ Now hours of work (or work schedules more generally) are treated as nonpecuniary aspects of jobs
 - ▷ Thus necessary to consider each work-hours opportunity as a separate labor market
- ▷ The market equilibrium equalizing differences function will take the form $W = W(H)$
 - ▷ The wage will be a function of hours demanded by the job
 - ▷ Goldin's convex wage function can be thus seen as a version of compensating wage differential!
- ▷ In equilibrium, workers with greater tastes for work are matched to firms and jobs for which productivity or cost reasons impel the firm to demand longer working hours

Regional inequality: motivation

- ▷ Observed intercity & regional wage differences among workers with similar productivity (as proxied by education & experience)
- ▷ It should be clear to you what will happen next but back in time while economists had recognized the locational amenities at consumer side, they didn't take into account the firm side
- ▷ **Roback (1982)**: if workers require a compensating wage differential to live in a big, polluted, or otherwise unpleasant city, the firms in that city must have some productivity advantage that allows them to pay the higher wage
- ▷ The Rosen-Roback framework has been extended widely and become a building block of modern spatial equilibrium models
- ▷ Key difference compared to Rosen's model
 - ▷ A spatial equilibrium in which both land and labor markets must clear
 - ▷ Utility equalization among locations show up both in wage differences and in differences in site-specific prices (rent)

Rosen-Roback model: setting

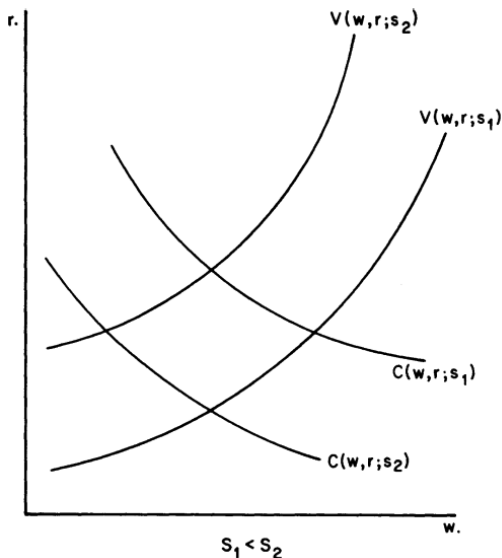
- ▷ Each city has a wage rate w , a price of land r , and a vector of amenities s (climate, pollution, crime, crowding, ...)
- ▷ Workers are identical and firms are identical; Workers and firms are perfectly mobile across cities (no moving costs);
- ▷ Lands are fixed (can be relaxed to be elastic)
- ▷ Worker's problem in each city: $\max_{x, l^c} U(x, l^c; s)$ s.t. $w = x + l^c r$
 - ▷ x is consumption on a national good ($p = 1$) and l^c is residential land
 - ▷ Because a higher level of s is preferred by workers, $\frac{\partial U}{\partial s} > 0$
 - ▷ We can rewrite an indirect utility function $V(w, r; s)$
- ▷ Firm's problem in each city: $\min C(X, w, r; s)$ s.t. $X = F(l^p, N; s)$
 - ▷ l^p is land used in production; N is the total number of workers in the city
 - ▷ Assume F is CRS, thus the unit cost of the representative firm is $C(w, r; s) = C/X = 1 = p$ (with some abuse of notation)
 - ▷ Amenity can be unproductive ($C_s > 0$; e.g. clean air) or productive ($C_s < 0$; e.g. less storms)

Rosen-Roback model: equilibrium

- ▷ In spatial equilibrium, individuals are indifferent across cities, i.e. wages and rents must adjust such that the indirect utility function $V(w, r; s) \equiv V^*$ is equalized across all cities
- ▷ Firms are also indifferent across cities, i.e. wages and rents must be such that the unit cost $C(w, r; s) = 1$ holds across all cities (which is already disciplined by the national product market)
- ▷ Indirect utility function and unit cost condition generate equilibrium levels of wages $w(s)$ and rents $r(s)$ for a given level of V^* , where V^* is determined by aggregate labor demand and aggregate labor supply

Rosen-Roback model: equilibrium ► derivation

In this case, the amenity s is unproductive; We have $w_1 > w_2$, $r_1 \geq r_2$



(Q: what if amenity is productive or neutral in production?)

Reference

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Appendix

The case with normal distribution

- ▷ Assume Z is normally distributed such that $Z = \bar{Z} + v$,
 $v \in \mathcal{N}(0, \sigma^2)$

$$n_1 = \text{Prob}(Z \leq \Delta W) = \text{Pr}(\bar{Z} + v \leq \Delta W)$$

- ▷ Proportion of workers:
$$= \text{Pr}\left(\frac{v}{\sigma} \leq \frac{\Delta W - \bar{Z}}{\sigma}\right) = \Phi\left(\frac{\Delta W - \bar{Z}}{\sigma}\right)$$

- ▷ Wage elasticity of labor supply:

$$\varepsilon_1 = \frac{\Delta W}{n_1} \frac{dn_1}{d(\Delta W)} = \left(\frac{\Delta W}{\sigma}\right) \left[\phi\left(\frac{\Delta W - \bar{Z}}{\sigma}\right) / \Phi\left(\frac{\Delta W - \bar{Z}}{\sigma}\right)\right]$$

- ▷ Conditional expectation:

$$\begin{aligned} E(Z \mid D = 1) &= E(\bar{Z} + v \mid \bar{Z} + v \leq \Delta W) = \bar{Z} + E(v \mid v \leq \Delta W - \bar{Z}) \\ &= \bar{Z} + \sigma E\left(\frac{v}{\sigma} \mid \frac{v}{\sigma} \leq \Delta W - \bar{Z}\right) \\ &= \bar{Z} - \sigma \phi\left(\frac{\Delta W - \bar{Z}}{\sigma}\right) / \Phi\left(\frac{\Delta W - \bar{Z}}{\sigma}\right) \end{aligned}$$

where the last equation uses **inverse Mills ratio**

The case of normal distribution (cont.)

- ▷ Selectivity bias:

$$S_1 = \bar{Z} - E(Z \mid D = 1) = \sigma \phi \left(\frac{\Delta W - \bar{Z}}{\sigma} \right) / \Phi \left(\frac{\Delta W - \bar{Z}}{\sigma} \right)$$

- ▷ Average rent:

$$R_1 = \Delta W - E(Z \mid D = 1) = \sigma \left[\frac{\Delta W - \bar{Z}}{\sigma} + \frac{\phi(\cdot)}{\Phi(\cdot)} \right] = \Delta w - \bar{Z} + S_1$$

- ▷ Average rent among $D = 1$ workers is thus increasing in Δw and in σ^2 ; As variance in preferences σ^2 goes to 0, R_1 goes to zero
- ▷ Note both ε_1 , S_1 and R_1 increases in the inverse Mill's ratio ϕ/Φ
- ▷ General point: information on conditional averages do not necessarily convey much information about decision-makers near the margin of choice, which determines the responsiveness of supply to relative wage movements

Human capital investment model with equalizing

- ▷ Consider a continuous-time human capital investment model
- ▷ A worker's flow earning: $y(t) = RK(t) - P(I(t))$,
 - ▷ R is the piece rate for human capital; $K(t)$ is the current stock of human capital
 - ▷ $I(t)$ is the learning opportunities chosen at t ; P is the market equalizing difference function
- ▷ Assume human capital investment function: $\dot{K} = dK/dt = \gamma I$, and the cost function $P(I) = I^2/2$
 - ▷ γ is interpreted as a learning efficiency parameter
- ▷ Thus $y(t) = RK(t) - [\dot{K}(t)/\gamma]^2/2$; The discounted present value is $\int_0^T e^{-rt} y(t) dt$
- ▷ FOC: $Q(t) \equiv \underbrace{(R/r) (1 - e^{-r(T-t)})}_{\text{(discounted) marginal return of hc invest}} = \underbrace{\dot{K}(t)/\gamma^2}_{\text{marginal cost of hc invest}}$
- ▷ $\Rightarrow \dot{K}^* = \gamma^2 Q(t); I^*(t) = \gamma Q(t)$

Rosen-Roback model: equilibrium

▷ Totally differentiating the indirect utility function $V(w, r; s) = V^*$ gives $v_w dw + v_r dr + v_s ds = 0$

▷ Totally differentiating the cost function $C(w, r; s) = 1 = p$ gives $c_w dw + c_r dr + c_s ds = 0$

▷ We can combine these equations to derive:

$$\frac{dw}{ds} = \frac{c_s v_r - c_r v_s}{c_r v_w - c_w v_r}$$
$$\frac{dr}{ds} = \frac{c_w v_s - c_s v_w}{c_r v_w - c_w v_r}$$

▷ The denominator is the same in both expressions: $c_r v_w - c_w v_r > 0$ (since $v_w > 0$, $c_r > 0$, $c_w > 0$, $v_r < 0$)

▷ $\frac{dw}{ds} < 0$ (since $c_s > 0$, $v_r < 0$, $c_r > 0$, $v_s > 0$), i.e. wages decline as s increases (since both worker and firm now require a smaller wage)

▷ $\frac{dr}{ds} \gtrless 0$ (since c_w , v_s , c_s , v_w all > 0) depends on the relative magnitude of $c_w v_s$ relative to $c_s v_w$ (since while worker now accepts a higher rent, firm now requires a lower rent)