Labor Demand: Technology and Inequality

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Fall, 2023

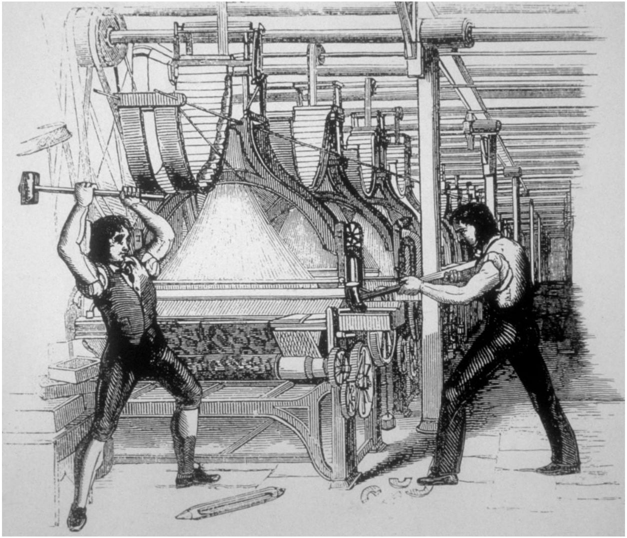
Roadmap

1. Introduction
2. Production and labor demand
3. Technology, labor supply, and skill premium
4. More on technological impact

Where do wages and jobs come from?

- In the labor supply part, we take wages as given, and workers always get their jobs as long as they wish to supply in the market
  - Also in the human capital theory, we take the fact that higher education yields higher productivity and higher wage as granted
- The missing part is the labor demand: firms hire workers to do the jobs and pay for their labor inputs
  - (Q: how about the era before the concept of ”firm” was even invented?)
- What factors affect modern labor demand?
- One most fundamental factor that economists found and studied is technology and its change over time
  - It has also be attributed to a major culprit behind wage inequalities

The Luddite Movement



”Why Are There Still So Many Jobs?”

- ▷ 1st IR (1800s) $\rightarrow$ ”Luddite Movement/Rebellion”
- ▷ 2nd IR (1930s) $\rightarrow$ ”Technological Unemployment”
- ▷ Post WW2 $\rightarrow$ ”The Automation Jobless”
- ▷ Recent AI Evolution $\rightarrow$ ”Taking Over 80% of Human Jobs”
- ▷ Why firms keep hiring workers to do the jobs?
- ▷ ”This time is different”?

Why firms pay highly educated workers more & more?

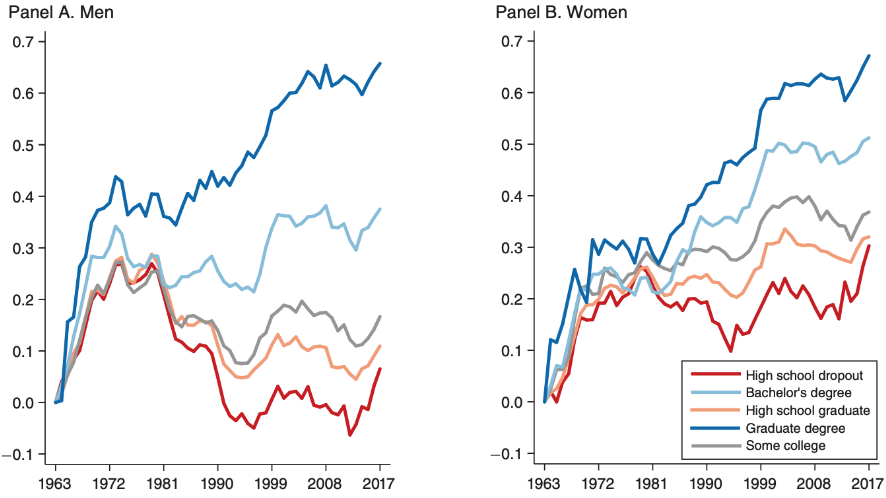


Figure 1. Cumulative Change in Real Weekly Earnings of Working-Age Adults Ages 18–64, 1963–2017

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Production, Labor Demand, and Market Equilibrium

▸ details

- Firm production function: $Y = F(K, L)$ ($F_K, F_L > 0, F_{KK}, F_{LL} < 0$)
- Assume F is constant return to scale (CRS) in K and L, i.e. $F(aK, aL) = aF(K, L)$ (allows to consider a representative firm and analyze aggregate demand)
- The firm problem: $\max_{L,K} pF(K, L) - wL - rK$ (normalize $p = 1$)
- FOCs/Trade-off: $F_K(K, L) = r$, $F_L(K, L) = w$
- Factor demand: $K^{demand} = K(w, r)$, $L^{demand} = L(w, r)$
- A competitive equilibrium requires $K^{demand}(r, w) = K^{supply}(r)$, $L^{demand}(r, w) = L^{supply}(w)$
    - If all factors (here K, L) are perfectly inelastically supplied (i.e. fixed), the aggregate labor demand pins down the wage
    - More generally, demand changes induce adjustments in both quantity and prices

Classic example: Cobb-Douglas

- Cobb-Douglas production function: $Y = F(K, L) = AK^{\alpha} L^{1-\alpha}$
  - (Q: Is this CRS? How can we make it DRS or IRS?)
- FOCs: $\alpha AK^{\alpha-1} L^{1-\alpha} = r$; $(1-\alpha)AK^{\alpha} L^{-\alpha} = w$
  - Note we can also write as $\alpha Y/K = r$; $(1-\alpha)Y/L = w$
- Marginal rate of (technical) substitution (MRS): $\frac{F_K}{F_L} = \frac{\alpha L}{(1-\alpha)K} = \frac{r}{w}$
  - MRS is only a function of the factor ratio K/L, which is a property of CRS production functions ‣ more technically homogenous or homothetic functions
- Note if labor supply is inelastic, technological advance (an increase in A) increases both r and w, but leaves r/w unchanged
- Using the FOCs, we can confirm that $Y = wL + rK$
  - Thus the firm has no profits, also a property of CRS
- Labor share: $\frac{wL}{Y} = \frac{F_K(K,L)K}{Y} = 1 - \alpha$
  - To see the factor share ratio, rewrite the MRS as $\frac{wL}{rK} = \frac{1-\alpha}{\alpha}$
  - This is why macroeconomists used to like CD form a lot: the aggregate labor share in an economy was regarded as constant in the long-run (though things have changed a little bit now) ‣ trend

Cobb-Douglas and Elasticity of Substitution

- Except being CRS and having fixed factor shares, there is another important feature of CD production function: its elasticity of substitution is constant and, in fact, 1

- **Elasticity of substitution (ES)**: $\sigma \equiv -\frac{\partial \ln(K/L)}{\partial \ln(F_K/F_L)} = -\left[\frac{\partial \ln(F_K/F_L)}{\partial \ln(K/L)}\right]^{-1}$
  - Since $\frac{F_K}{F_L}$ is the slope of the isoquant (for $F(K,L) = Y$), σ is the proportional change of the relative use of the two factors per percent change in the slope of the isoquant
  - Intuition: it measures how easily one input can be substituted for another

- Note that under competitive market, we also have $\sigma \equiv -\frac{\partial \ln(K/L)}{\partial \ln(F_K/F_L)} = -\frac{\partial \ln(K/L)}{\partial \ln r/w}$ (which is perhaps more intuitive)

- In the CD case, $\sigma = -\left[\frac{\partial \ln \frac{\alpha L}{(1-\alpha)K}}{\partial \ln(K/L)}\right]^{-1} = 1$
  - This is in fact why the factor shares are fixed with CD form
  - (Q: what's the interpretation when $\sigma = 0$ or $\sigma = \infty$?)

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The Canonical Model of Skill Differential

- The simplest framework for interpreting skill premia (e.g. returns to schooling or other skills)
- A competitive supply-demand framework in a simple closed economy setting (i.e. no international trade)
- Factors are paid their marginal products
- Relax the restrictions in CD form:
  - Technology does not affect relative factor prices
  - Fixed shares paid to each factor
- A ”workhorse” model which has been enriched in many ways

CES aggregate production function

- $Y = \left[(A_l L)^\rho + (A_h H)^\rho\right]^{1/\rho}$, where $\rho \leq 1$
  - L, H are two types of workers, skilled/unskilled (high/low education, college/non-college, etc.)
  - A_l, A_h are factor-specific technologies, compared to the Hicks-neutral technology in CD form
- MRS: $\frac{\partial Y/\partial L}{\partial Y/\partial H} = \frac{A_l^\rho (Y/L)^{1-\rho}}{A_h^\rho (Y/H)^{1-\rho}}$; ES: $\sigma = 1/(1-\rho) \geq 0$ (hence CES)
- If $\sigma > 1$ (or $\rho > 0$): "gross substitutes"; If $\sigma \to \infty$ (or $\rho \to 1$): "perfect substitutes" as $Y = A_l L + A_h H$ (linear)
- If $\sigma < 1$ (or $\rho < 0$): "gross complements"; If $\sigma \to 0$ (or $\rho \to -\infty$): "perfect complements" as $Y = \min\{A_l L, A_h H\}$ (Leontief) derivation
- If $\sigma \to 1$ (or $\rho \to 0$): $Y = (A_l L)^{\frac{1}{2}} (A_h H)^{\frac{1}{2}}$ (Cobb-Douglas) derivation
- ES is critical because it determines how changes in either technology (A_l, A_h) or labor supplies (L, H) affect demand & wages

Wage Determination

- $w_L = \frac{\partial Y}{\partial L} = A_l^\rho \left[A_l^\rho + A_h^\rho (H/L)^\rho\right]^{(1-\rho)/\rho}$
  $w_H = \frac{\partial Y}{\partial H} = A_h^\rho \left[A_h^\rho + A_l^\rho (H/L)^{-\rho}\right]^{(1-\rho)/\rho}$
- $\partial w_H / \partial (H/L) \propto (\rho - 1) \leq 0$: as fraction of skilled workers in labor force increases, the wages of skilled workers should decrease (own labor demand curve is downward sloping)
- $\partial w_L / \partial (H/L) \propto (1 - \rho) \geq 0$: as fraction of skilled workers in labor force increases, the wages of unskilled workers should increase
- When $\rho \to 1$ ($\sigma \to \infty$), both derivatives are 0 as two types of workers are perfect substitutes
- When $\rho \to -\infty$ ($\sigma \to 0$), both effects are infinitely large
- Note our assumption $\rho \leq 1$ ($\sigma \geq 0$) in fact ensures "Q-complements" or "Supermodularity": a greater quantity of the one increases marginal product of the other (i.e. $\frac{\partial^2 Y}{\partial L \partial H} > 0$)

Wage Premium and Labor Supply

- $\omega = \frac{w_H}{w_L} = \left(\frac{A_h}{A_l}\right)^{\rho} \left(\frac{H}{L}\right)^{-(1-\rho)} = \left(\frac{A_h}{A_l}\right)^{(\sigma-1)/\sigma} \left(\frac{H}{L}\right)^{-1/\sigma}$

- $\ln \omega = \left(\frac{\sigma-1}{\sigma}\right) \ln \left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma} \ln \left(\frac{H}{L}\right)$

- $\frac{\partial \ln \omega}{\partial \ln(H/L)} = -\frac{1}{\sigma} < 0$, i.e. for given skill bias A_h/A_l, an increase in relative supplies H/L lower relative wages with elasticity $1/\sigma$
  - Intuition: more tasks being allocated to L from H, decreasing marginal product of H and increasing marginal product of L
  - (Q: why a larger σ yields a smaller effect?)

- (Q: is this a good news for children born in the baby boom or for students during an college education expansion?)

- The estimates in the literature indicate $\sigma \in (1.4, 2)$ a classic estimate

College Premium & College-graduate Supply in Japan

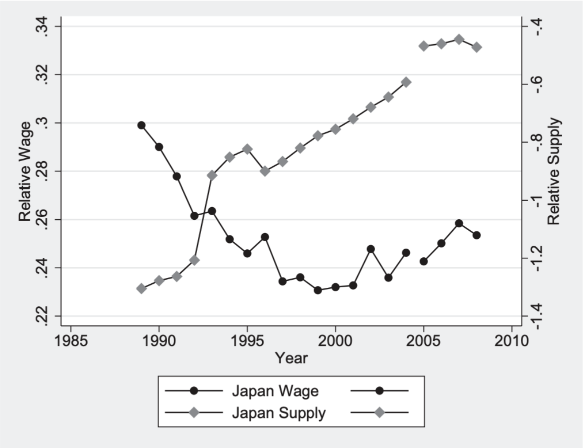

Fig. 5. Quantity of 4-year-college-graduate workers and the college wage premium, males 25–59, 1989–2006, Japan (BSWS). *Note:* Relative wages are calculated using male hourly wages. The supply measure is calculated based on male workers. The relative supply of college-educated workers to high-school-educated workers refers to the log (total hours worked by college-educated workers/total hours worked by high-school-educated workers). Hours worked by junior- or technical-college graduates are prorated to hours worked by college-educated or high-school-educated workers using the average hourly wage rates of the sample period as the weights for prorating.

Wage Premium and Technological Change

- $\ln \omega = \left(\frac{\sigma-1}{\sigma}\right) \ln\left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma} \ln\left(\frac{H}{L}\right)$
- $\frac{\partial \ln \omega}{\partial \ln(A_h/A_l)} = \frac{\sigma-1}{\sigma} \lesseqgtr 0$, i.e. the sign depends upon $\sigma \lesseqgtr 1$
- Why a rise in the productivity of skilled relative to unskilled (A_h/A_l) will causes the skill wage premium to fall (when $\sigma < 1$)?
  - Intuition: an increase in supply of high skilled workers effectively creates "excess supply" for a given number of unskilled workers
  - However since the broad consensus is $\sigma > 1$, this case is generally thought to be unlikely
- If $\sigma > 1$, $\frac{\partial \ln \omega}{\partial \ln(A_h/A_l)} > 0$, and we now have a reason for the increased college premium even with college expansion: an increase in A_h/A_l (i.e. "skill-biased technological change")
  - (Q: what are the examples of such technological advance? is there any technologies go the other way?)
- The two forces of increased schooling (H/L) and technological development (A_h/A_l) have been summarized as "a Race between Education and Technology"

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A problem of the factor-augmenting CES model

- If A_h or A_l rises with $\sigma > 1$, wages should rise for all workers, both skilled and unskilled (though inequality may increase)
  - This is a result of Q-complements
  - Factor augmenting technical change thus always raises societal wealth since we can get more output for a given set of inputs
- But wages of non-college men fell substantially in real terms in U.S. during the 1980s and in some other industrialized economies; In the meantime, non-college workers have become relatively scarce
- Our CES model cannot account for this wage falling unless we wish to argue that A_l falls, but why should there be any technological regress?

Task-based framework and Automation

- Acemoglu and Restrepo (2018a,b, 2019) suggests that factor-augmenting technological change does not correctly capture automation-like technological changes
- An abstract sketch of task-based framework see the original model: $Y = \left[\alpha \left(A_k K\right)^\rho + (\beta - \alpha)\left(A_l L\right)^\rho + (1-\beta)\left(A_h H\right)^\rho\right]^{1/\rho}$
  - Production is achieved by completing a serious of tasks, with the shares being done by three types of factor (K, L, H) defined by two thresholds α and β ($0 < \alpha < \beta < 1$)
  - Assume $A_k > A_l$, thus machine is more efficient for the tasks done by L if technology is available
- With $\rho \to 0$, $Y = AK^\alpha L^{\beta-\alpha} H^{1-\beta}$, where $A \equiv A_k^\alpha A_l^{\beta-\alpha} A_h^{1-\beta}$
- Recall with CD form, $rK = \alpha Y$, $w_l L = (\beta - \alpha) Y$, $w_h H = (1-\beta) Y$

Displacement Effect, Productivity Effect, Capital Deepening, and Reinstatement Effect

- $w_l = (\beta - \alpha) Y/L$, $w_h = (1 - \beta) Y/H$
- AR suggests that automation is an increase in α, i.e. the range of tasks can be conducted by machines
- An increase in α generates a direct and negative "displacement effect" for labor type L through $(\beta - \alpha)$
- Because machine is more efficient, there is also a counteracting and positive "productivity effect" through Y/L
- If displacement effect is dominated, w_l can decline; In contrast, w_h always increases as there is only positive productivity effect
- An increase in A_k ("capital deepening") increases both w_l and w_h
- Creation of new tasks that only human can do ("reinstatement effect") generates the opposite effect of displacement examples

Who had been replaced in the last several decades?

- The recent technological advance had been the diffusion of computer-based technologies
- Autor et al. (2003) considers two questions: ”what tasks computers do?” ”what tasks human do?”
- They argue that computers do ”routine codifiable” tasks
  - Computers ”rapidly and accurately perform repetitive tasks that are deterministically specified by stored instructions (programs) that designate unambiguously what actions the machine will perform at each contingency”
  - Activities ”that can be fully described by a set of rules and procedures, encoded in software, and carried out by nonsentient machines”
- They thus suggest computer capital see examples of each category
  - Substitutes workers doing ”routine tasks”: repetitive and well-defined set of cognitive and manual activities
  - Complements workers doing ”non-routine tasks”: creative, abstract, problem-solving, and communicating activities (i.e. tasks are not well described by a tightly specified scripts that machines can execute)

Who will be replaced in the future?

- This previous dichotomy (routine vs non-routine) has been overtuned under recent (and potentially future) AI technologies
  - Those ”non-routine” tasks have proven hard to automate because, simply put, ”we don't know the rules” (i.e. ”tacit” knowledge)
  - AI tools surmount this longstanding constraint because they can be used to infer tacit relationships that are not fully specified ▸ in math
- The best answer so far is ”we don't know” (Autor, 2022) or ”an empirical question”; But task framework still provides a useful starting point
  - To what extent and for what working tasks will AI prove capable of accomplishing in the years (and decades) ahead?
  - To what extent and for what new demands for human skills and capabilities will emerge as AI displaces a growing set of traditional human work tasks?
  - There are some preliminary results on AI improving the productivity of either the high-skilled or the low-skilled, as well as on AI jobs replacing non-AI jobs at firm level but not at any aggregate level

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Appendix

Production Function

- Firm production function: $Y = F(K, L)$
  - Assume F is increasing and diminishing return to both arguments, i.e. $F_K > 0, F_L > 0$, and $F_{KK} < 0, F_{LL} < 0$ (intuition: positive and diminishing marginal products)

- We often assume F is constant return to scale (CRS) in K and L, i.e. $F(aK, aL) = aF(K, L)$ ▸ CRS $\equiv$ Homogenous of Degree 1
  - This means in a competitive market the scale of individual firms is undefined $\rightarrow$ firms per se are unimportant, and, if all firms have the same production function, we can consider a representative firm and analyze the aggregate demand
  - Some useful property under CRS: $F = F_K K + F_L L$, and $F_K(aK, aL) = F_K(K, L)$, $F_L(aK, aL) = F_L(K, L)$ ▸ Euler's Theorem

- F being decreasing return to scale (DRS) along with heterogeneous firms are sometimes used when studying the entire distribution of firm size and input use is important (e.g. IO or trade) ▸ more details
  - (Q: why increasing return to scale (IRS) is rarely considered?)

Firm Optimization

- Assume both product market and labor market are perfectly competitive (i.e. price-taking; relax next week)
- The firm problem: $\max_{L,K} pF(K, L) - wL - rK$
  - Competitive product market allows us to normalize the product price p to 1
  - Note that the way we write prices p, w, and r already imposes the price-taking nature of perfectly competitive markets
- FOCs/Trade-off: $F_K(K, L) = r$, $F_L(K, L) = w$
- Factor demand: $K = K(w, r)$, $L = L(w, r)$
- Because F exhibits CRS, the problem does not have a well-defined solution (Q: can you see this from FOCs?)
  - Related to the fact that in a world with CRS, the size of each individual firm is not determinate
  - Only aggregates are determined after imposing the condition that factor markets should clear

Market Equilibrium

- A competitive equilibrium requires that all firms (and thus the RF) maximize profits and factor markets clear
  - Factor markets clear means demands for labor and capital must be equal to the supplies of these factors ($K^{demand} = K^{supply}$, $L^{demand} = L^{supply}$), which are functions of market prices/wages
- Today we abstract from the supply side problem and assume all factors (here K, L) are perfectly inelastically supplied
  - Thus factors use are fixed, and the aggregate labor demand pins down the wage
  - However, since the perfectly competitive factor markets, each individual firm faces perfectly elastic labor supply and thus takes the market price as given
- All firms make zero profits: $Y = wL + rK$ (Q: how to obtain this?)
  - If e.g. demand $L < L(t)$, then there would be an excess supply of labor and the wage would be equal to zero. But this is not consistent with firm maximization, since RF would then wish to hire an arbitrarily large amount of labor, exceeding the supply

Homogeneity

- Definition: Let $K \in \mathbb{N}$. The function $g : \mathbb{R}^{K+2} \to \mathbb{R}$ is Homogenous of Degree (HD) m in $x \in \mathbb{R}$ and $y \in \mathbb{R}$ if $g(\lambda x, \lambda y, z) = \lambda^m g(x, y, z)$ for all $\lambda \in \mathbb{R}_+$, $z \in \mathbb{R}^K$
- Note with CRS (HD1 or linear homogeneity) and $F_{KK}, F_{LL} < 0$, the the production function $F(K, L)$ is concave, though not strictly so
  - To see why, recall concavity for multivariate function requires the Hessian matrix $H = \begin{bmatrix} F_{KK} & F_{KL} \\ F_{KL} & F_{LL} \end{bmatrix}$ to be semi-negative definite, i.e. $F_{kKK}, F_{LL} \leq 0$ & $F_{KK}F_{LL} - F_{KL}^2 \geq 0$ (first principle minor be non-positive & second principle minor be non-negative)
  - Use the Euler's Theorem (see next slide), $F(K, L) = KF_K + LF_L \Rightarrow$ $F_L = KF_{KL} + LF_{LL} + F_L, \; F_K = LF_{KL} + KF_{KK} + F_K$
  - Substituting F_{LL}, F_{KK} into the second principle minor: $\left(-\frac{L}{K}F_{KL}\right)\left(-\frac{K}{L}F_{KL}\right) - F_{KL}^2 = 0$

Euler's Theorem

- Euler's Theorem: Suppose $g : \mathbb{R}^{K+2} \to \mathbb{R}$ is differentiable in $x \in \mathbb{R}$ and $y \in \mathbb{R}$, with partial derivatives g_x, g_y, and is HD m in x and y.
  - Then $mg(x, y, z) = g_x(x, y, z)x + g_y(x, y, z)y$ for all $x \in \mathbb{R}$, $y \in \mathbb{R}$, and $z \in \mathbb{R}^K$.
  - Moreover, $g_x(x, y, z)$ and $g_y(x, y, z)$ are themselves HD $m - 1$ in x and y.

- Proof. We have $\lambda^m g(x, y, z) = g(\lambda x, \lambda y, z)$.
  - Differentiate both sides with respect to λ: $m\lambda^{m-1} g(x, y, z) = g_x(\lambda x, \lambda y, z)x + g_y(\lambda x, \lambda y, z)y$ for any λ. Setting $\lambda = 1$ yields the first result.
  - Differentiate both sides with respect to x: $\lambda g_x(\lambda x, \lambda y, z) = \lambda^m g_x(x, y, z)$. Dividing both sides by λ establishes the second result.

Decreasing Return to Scale (DRS)

- In some cases, we will assume F is decreasing return to scale (DRS) and allows heterogenous production function, e.g. $F_i(L) = z_i L_i^{\alpha}$ where z_i is firm's idiosyncratic productivity and $0 < \alpha < 1$ controls the decreasing return to scale
- In such cases, each firm's downward-slopping marginal product curve helps to pin down its scale, allowing us to study the entire distribution of firm size and firm input use
- However, such curvature can be also obtained through demand system (e.g. CES demand aggregator) while retaining CRS, as eventually what we want is the marginal revenue product curve to be donward-slopping

Homothetic production function

- A production function is homothetic if it can be represented as a monotonic transformation of a homogeneous function. That is, $G(K, L) = h(F(K, L))$, where h is a monotonic transformation, and F is a homogeneous function
- The MRS is thus $\frac{h' F_K}{h' F_L} = \frac{F_K}{F_L}$
- Recall for homogenous of degree m function we have $F_K(\lambda K, \lambda L) = \lambda^{m-1} F_K(K, L), F_L(\lambda K, \lambda L) = \lambda^{m-1} F_L(K, L)$
- Thus we have $F_K(K/L, 1) = L^{1-m} F_K(K, L), F_L(K/L, 1) = L^{1-m} F_L(K, L)$
- Substitute into MRS: $\frac{F_K}{F_L} = \frac{F_K(K/L,1)L^{m-1}}{F_L(K/L,1)L^{m-1}} = \frac{F_K(K/L,1)}{F_L(K/L,1)}$ for which we can define a new function $H(K/L)$ which is solely a function of the ratio of the factors

Labor Share (US)

(A huge literature has studied the recent labor share decline, with little consensus)

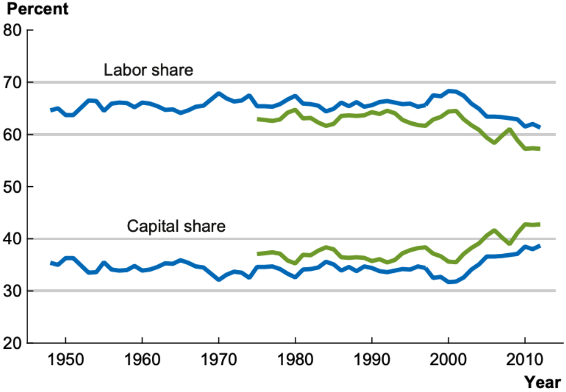


Fig. 6 Capital and labor shares of factor payments, United States. Source: *The series starting in 1975 are from Karabarbounis, L., Neiman, B. 2014. The global decline of the labor share. Q. J. Econ. 129 (1), 61–103. http://ideas.repec.org/a/oup/qjecon/v129y2014i1p61-103.html and measure the factor shares for the corporate sector, which the authors argue is helpful in eliminating issues related to self-employment. The series starting in 1948 is from the Bureau of Labor Statistics* Multifactor Productivity Trends, *August 21, 2014, for the private business sector. The factor shares add to 100%.*

CES with $\rho \to -\infty$

- Consider the general case: $Y = Z\left[b\left(A_l L\right)^{\rho} + (1-b)\left(A_h H\right)^{\rho}\right]^{1/\rho}$

- Since we are interested in the limit when $\rho \to -\infty$ we can ignore the interval for which $\rho \geq 0$, and treat ρ as strictly negative

- First assume $A_l L \leq A_h H \Rightarrow (A_l L)^{\rho} \geq (A_h H)^{\rho}$

- Then we verify that the following inequality holds:
$$(1-b)^{1/\rho} A_l L \geq Y/Z \geq A_l L$$
$$\Rightarrow (1-b)^{1/\rho} A_l L \geq \left[b\left(A_l L\right)^{\rho} + (1-b)\left(A_h H\right)^{\rho}\right]^{\frac{1}{\rho}} \geq A_l L$$
$$\Rightarrow (1-b)\left(A_l L\right)^{\rho} \leq b\left(A_l L\right)^{\rho} + (1-b)\left(A_h H\right)^{\rho} \leq \left(A_l L\right)^{\rho}$$

- Then given that $\lim_{\rho \to -\infty}(1-b)^{1/\rho}\left(A_l L\right) = A_l L$, the middle term Y/Z in the inequality is sandwiched, so $\lim_{\rho \to -\infty} Y/Z = A_l L$

- Similarly when $A_l L \geq A_h H \Rightarrow \lim_{\rho \to -\infty} Y/Z = A_h H$

- Combine two cases, we have $\lim_{\rho \to -\infty} Y = Z \min\{A_l L, A_h H\}$

CES with $\rho \to 0$

- Consider the general case: $Y = Z\left[b\left(A_l L\right)^{\rho} + (1-b)\left(A_h H\right)^{\rho}\right]^{1/\rho}$
- We want to evaluate $\lim_{\rho\to 0} Y$ but it will turn out to be easier to evaluate $\lim_{\rho\to 0} \ln(Y/Z)$
- $\lim_{\rho\to 0} \ln(Y/Z) = \lim_{\rho\to 0} \frac{\ln\left[b(A_l L)^{\rho} + (1-b)(A_h H)^{\rho}\right]}{\rho}$
- As this evaluates to $\frac{0}{0}$ we use L'Hospital's rule:

$$\begin{aligned}\lim_{\rho\to 0} \ln(Y/Z) &= \lim_{\rho\to 0} \frac{b\left(A_l L\right)^{\rho}\ln(A_l L) + (1-b)\left(A_h H\right)^{\rho}\ln(A_h H)}{b\left(A_l L\right)^{\rho} + (1-b)\left(A_h H\right)^{\rho}} \\ &= \frac{b\ln(A_l L) + (1-b)\ln(A_h H)}{b + (1-b)}\end{aligned}$$

- Taking exponents and multiplying Z gives: $\lim_{\rho\to 0} Y = Z(A_l L)^{b}(A_h H)^{1-b}$
- The simplified form in main text is when $b = 1/2, Z = (1/2)^{-1/\rho}$

Summary of CES model results

- In response to an increase in H/L
  - Skill premium $\omega = W_H/W_L$ falls
  - W_L (for unskilled) rise and W_H (for skilled) decrease
  - Average wages $\bar{w} = \frac{LW_L + HW_H}{L+H} = \frac{\left[(A_l)^\rho + (A_h H/L)^\rho\right]^{1/\rho}}{1+H/L}$ rise provided skill premium is positive $\left(\omega > 1 \text{ or } A_h^\rho (H/L)^\rho - A_l^\rho > 0\right)$
- In response to an increase in A_h, holding A_l and L/H constant
  - $\omega = W_H/W_L$ rises if $\sigma > 1$, falls if $\sigma < 1$, and is unchanged if $\sigma = 1$
  - W_L rise if $\sigma < \infty$
  - Average wages rise if $\sigma > 0$
  - Both W_H and W_L rise if $\sigma \geq 1$
- Note that these results can readily be generalized to with capital, i.e., $F(A_l L, A_h H, K)$
  - But depending on the assumptions on capital supply and capital-skill complementarity, the specific predictions may differ

Bringing the CES model to the data

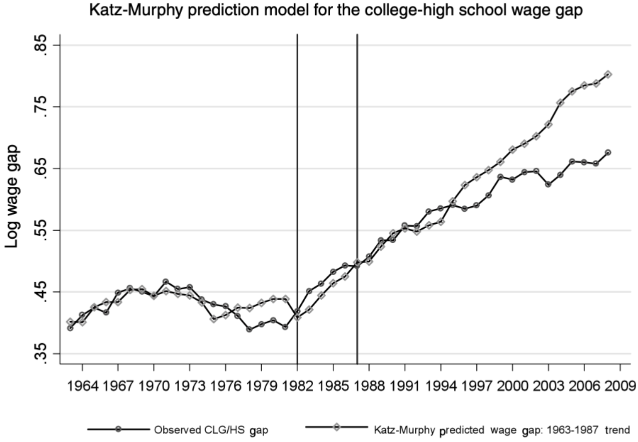
- $\ln \omega = \left(\frac{\sigma-1}{\sigma}\right) \ln \left(\frac{A_h}{A_l}\right) - \frac{1}{\sigma} \ln \left(\frac{H}{L}\right)$
- We have data in ω and H/L, and we want to estimate σ and A_h/A_L
- We assume $\left(\frac{\sigma-1}{\sigma}\right) \ln (A_h/A_l)_t = \gamma_1 t$, where t is the year index
- So we can estimate this model as:
  $\ln \omega_t = \gamma_0 + \gamma_1 t + \gamma_2 \ln(H/L) + e_t$, where $\hat{\gamma}_2$ is an estimate of $-\frac{1}{\sigma}$
- Using US data between 1963-1987, Katz and Murphy (1992) fit this model using a simple OLS regression:

$$\ln \omega = \underset{(0.01)}{0.033 \cdot t} \quad \underset{(0.15)}{-0.71 \cdot \ln \left(\frac{H}{L}\right)} + \text{constant}$$

  - There has been a trend increase in the relative demand for skilled worker
  - ES between them $\widehat{\sigma} = -1/0.709 = 1.41$

Can the estimated model of KM predict future?

Acemoglu and Autor (2011) projects the KM estimates (from 1963-1987 data) forward to 2008, showing that KM model continues to fit the aggregate data extremely well to 1995 but goes somewhat awry after that, arguably implying that demand growth from technological advance decelerates if assuming σ is constant



Task-based framework and Automation

- Here we show a simplified version of the task-based framework designed in Acemoglu and Restrepo (2018a)

- Aggregate output: $\ln Y = \int_{N-1}^{N} \ln y(x) dx$, where $x \in [N-1, N]$
  - Note this is just a continuous version of Cobb-Douglas production function; For the analysis with a CES one see Acemoglu and Restrepo (2018b)

- Task production:

$$y(x) = \begin{cases} A_L(x)\ell(x) + A_M(x)m(x) & \text{if } x \in [N-1, I] \\ A_L(x)\ell(x) & \text{if } x \in (I, N] \end{cases}$$

  - Tasks $x \in [N-1, I]$ are technologically automated
  - $A_L(x)/A_M(x)$ is increasing in x, and thus labor has a comparative advantage in higher-indexed tasks

- Aggregate output (GDP) in the equilibrium takes the form

$$Y = B\left(\frac{K}{I-N+1}\right)^{I-N+1}\left(\frac{L}{N-I}\right)^{N-I}, \text{ where}$$

$$B = \exp\left(\int_{N-1}^{I} \ln A_M(x) dx + \int_{I}^{N} \ln A_L(x) dx\right)$$ ▸ derivation

Displacement Effect vs. Productivity Effect

- $Y = B\left(\frac{K}{I-N+1}\right)^{I-N+1}\left(\frac{L}{N-I}\right)^{N-I}$ is just the Cobb-Douglas case
- Thus we have $RK = Y(I-N+1)$, $WL = Y(N-I)$
- Note factor augmenting parameters (A_L, A_M) only affect B
- $\frac{d\ln W}{dI} = \underbrace{\frac{d\ln(N-I)}{dI}}_{\text{Displacement effect } <0} + \underbrace{\frac{d\ln(Y/L)}{dI}}_{\text{Productivity effect } >0}$
  - Displacement effect: automation expands the set of tasks that capital/machine can do, and if capital is sufficiently cheap, then automation will lead to the substitution of capital for labor in these tasks, causing a decline in demand for labor
  - Productivity effect: by reducing the cost of producing a subset of tasks, automation raises demand for other non-automated tasks, causing an increase in demand for labor doing these tasks
- So far assumed a fixed supply of capital, if we allow capital be perfectly elastic (in long-run), productivity effect dominates

Two more "good" effects

- Deepening of Automation ($A_M(x) \uparrow$ for $x \in [N-1, I]$):
  - Improvements in already-existing automation machines or newer, more productive vintages will not create additional displacement but generate productivity effects
  - To see this, if we assume $A_M(x) = A_M$ in all automated tasks, we have $\frac{d \ln W}{\ln A_M} = \frac{d \ln Y/L}{\ln A_M} = (I - N + 1) > 0$

- Creation of new tasks / Reinstatement effect ($N \uparrow$)
  - Intensive automation and technological development have often coincided with the emergence of new jobs, activities, and tasks that only human labor can conduct
  - It engenders both a productivity effect and a reinstatement effect (which is just the opposite effect of displacement):

$$\frac{d \ln W}{dN} = \underbrace{\ln\left(\frac{R}{A_M(N-1)}\right) - \ln\left(\frac{W}{A_L(N)}\right)}_{\text{Productivity effect}>0} + \underbrace{\frac{1}{N-I}}_{\text{Reinstatement effec }>0}$$

Derive task-based production function

- Price of task: $p(x) = \begin{cases} \frac{R}{A_M(x)} & \text{if } x \in [N-1, I] \\ \frac{W}{A_L(x)} & \text{if } x \in (I, N] \end{cases}$
- Demand for task: $y(x) = \frac{Y}{p(x)}$
- Demand for machines: $k(x) = \begin{cases} \frac{Y}{R} & \text{if } x \in [N-1, I] \\ 0 & \text{if } x \in (I, N] \end{cases}$,
- Demand for labor: $\ell(x) = \begin{cases} 0 & \text{if } x \in [N-1, I] \\ \frac{Y}{W} & \text{if } x \in (I, N] \end{cases}$
- Aggregating demand and following market-clearing (with inelastic supply of K and L): $K = \frac{Y}{R}(I - N + 1)$; $L = \frac{Y}{W}(N - I)$
- Equilibrium rental rate & wage: $R = \frac{Y}{K}(I - N + 1)$, $W = \frac{Y}{L}(N - I)$

Derive task-based production function (cont.)

- With final good price normalized: $\int_{N-1}^{N} \ln p(x) dx = 0$ thus $\int_{N-1}^{I} [\ln R - \ln A_M(x)]\, dx + \int_{I}^{N} [\ln W - \ln A_L(x)]\, dx = 0$
- Thus
$$\begin{aligned} &\int_{N-1}^{I} [\ln Y - \ln(K/(I-N+1)) - \ln A_M(x)]\, dx \\ &+ \int_{I}^{N} [\ln Y - \ln(L/(N-I)) - \ln A_L(x)]\, dx = 0 \end{aligned}$$
- Rearrange
$$\begin{aligned} \ln Y &= \int_{N-1}^{I} \left[\ln\left(\frac{K}{I-N+1}\right) + \ln A_M(x)\right] dx + \int_{I}^{N} \left[\ln\left(\frac{L}{N-1}\right) + \ln A \right. \\ &= \int_{N-1}^{I} \ln A_M(x) dx + \int_{I}^{N} \ln A_L(x) dx \\ &\quad + (I-N+1) \ln\left(\frac{K}{I-N+1}\right) + (N-I) \ln\left(\frac{L}{N-I}\right), \end{aligned}$$

New born occupations

YEAR	EXAMPLE OF TITLES ADDED	
1940	Automatic welding machine operator	Gambling dealer
1950	Airplane designer	Beautician
1960	Textile chemist	Pageants director
1970	Engineer computer application	Mental-health counselor
1980	Controller, remotely piloted vehicle	Hypnotherapist
1990	Certified medical technician	Conference planner
2000	Artificial intelligence specialist	Chat room host/monitor
2010	Wind turbine technician	Sommelier
2018	Pediatric vascular surgeon	Drama therapist

Table 1: Examples of new occupational titles added to the U.S. Census Bureau's Classified Index of Occupations between 1940 and 2018 Source:

Autor et al. (2021b).

Routine vs. Non-Routine tasks

TABLE I

PREDICTIONS OF TASK MODEL FOR THE IMPACT OF COMPUTERIZATION ON FOUR CATEGORIES OF WORKPLACE TASKS

	Routine tasks	Nonroutine tasks
	Analytic and interactive tasks	
Examples	• Record-keeping • Calculation • Repetitive customer service (e.g., bank teller)	• Forming/testing hypotheses • Medical diagnosis • Legal writing • Persuading/selling • Managing others
Computer impact	• Substantial substitution	• Strong complementarities
	Manual tasks	
Examples	• Picking or sorting • Repetitive assembly	• Janitorial services • Truck driving
Computer impact	• Substantial substitution	• Limited opportunities for substitution or complementarity

Routine Exposure and Wage Changes

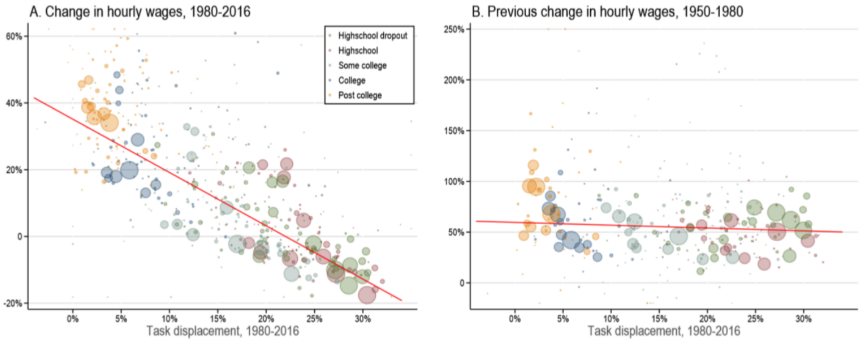


Figure 5: Exposure to task displacement and changes in real wages by demographic group, United States, 1980–2016 and 1950–1980

Source: Acemoglu and Restrepo (2021).

Note: Each marker corresponds to one of 500 demographic groups, defined by gender, age, education, race, and native/immigrant status. Marker sizes indicate the share of hours worked by each group and different colors indicate education levels.

Machine Learning is all about ”learning” (approximating) an unknown function: $y = f(\mathbf{x})$