Labor Supply: Family and Gender

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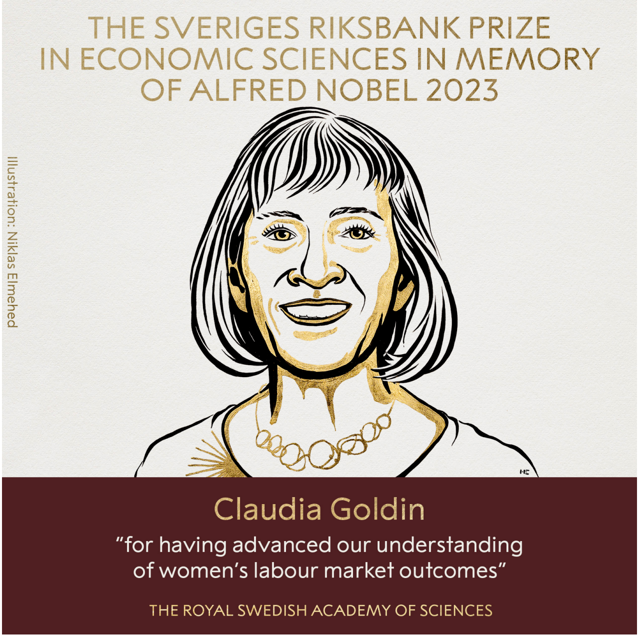
Roadmap

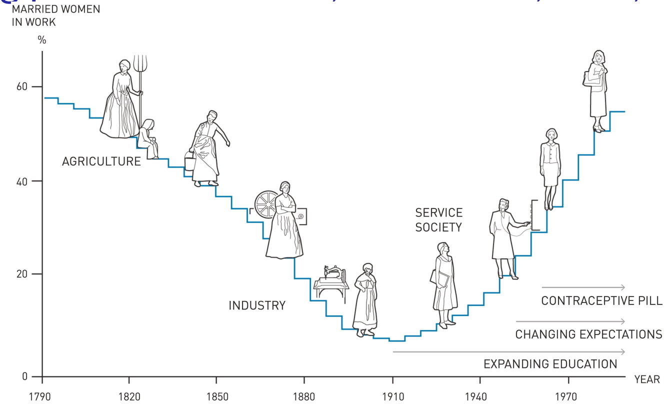
1. Introduction
2. Family Model
3. Explain female labor supply
4. Beyond unidimensional FLP

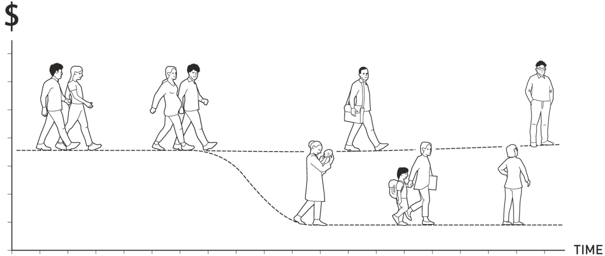
Introduction



Missing pieces: household, housework, child, ...

▸ history

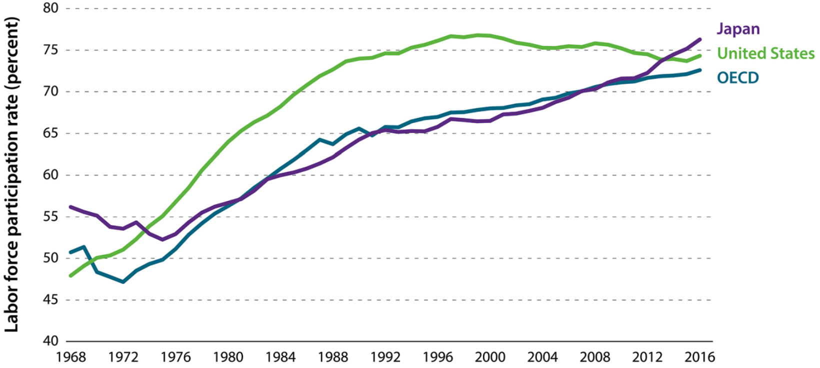




FLFP increase is important, but there are more

FIGURE 1.

Labor Force Participation of Prime-age Women from 1968–2016, by Country



Source: OECD Labour Force Statistics.
Note: Prime-age indicates 25–54.

THE HAMILTON PROJECT

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Unitary Model

- A family or household **utility**: $U(C, L_1, L_2)$
  - Note we consider consumption measured as a single aggregate, i.e. $C = C_1 + C_2$ and C_1 and C_2 can be decided arbitrarily
  - L_1 and L_2 can include both leisure and housework time

- Budget constraint: $C = Y + W_1(T - L_1) + W_2(T - L_2)$, where $Y = Y_1 + Y_2$

- FOCs:
$$U_C(C, L_1, L_2) = \lambda$$
$$U_{L1}(C, L_1, L_2) = \lambda W_1$$
$$U_{L2}(C, L_1, L_2) \geq \lambda W_2$$

- **Tradeoff**: $U_{L1} = U_C W_1$; $U_{L2} \geq U_C W_2$

- Demand function:
$$C = C(W_1, W_2, Y)$$
$$L_1 = L_1(W_1, W_2, Y)$$
$$L_2 = L_2(W_1, W_2, Y)$$

Restriction: No "Individualism"

other restrictions

- In unitary model, couples have preferences and incentives perfectly aligned
- In reality, a family can be a collection of individuals with their own preferences, e.g. "Battle of the sexes" in game thoery
- So there can have "egoistic" individual utilities $U_1(C_1, L_1)$ and $U_2(C_2, L_2)$
  - Now all goods are privately consumed i.e. private goods
- But there can also have more "altruistic" and general forms: $U_1(C_1, L_1, C_2, L_2)$ and $U_2(C_2, L_2, C_1, L_1)$
  - Less generally, there is also a "caring" type utility function: $W_1(U_1(C_1, L_1), U_2(C_2, L_2))$ and $W_2(U_2(C_2, L_2), U_1(C_1, L_1))$
- Family members can be either cooperative (e.g. bargaining) or noncooperative (when?) on how their optimization interacts
- It is thus potentially interesting to study how various way of interactions between family members impact their behaviors

Collective Models

- Collective models: individuals have utility functions + households as economic environments
- Cooperative model typically indicates Pareto efficiency
- (Efficient) Family problem: $\max\left[\theta U_1 + (1-\theta) U_2\right]$ s.t. $C_1 + C_2 = W_1(T - L_1) + W_2(T - L_2) + Y_1 + Y_2$
  - $\theta \in (0, 1)$ is the utility weight (or Pareto weight) for person 1, given by some function $\theta = f(W_1, W_2, Y_1, Y_2)$ and often interpreted as bargaining power (with the bargaining process not-specified here)
- If each member's utility is egoistic or caring type (i.e. separability), this family problem is equivalent to the decentralized problems: $\max U_1$ s.t. $C_1 = W_1(T - L_1) + Y - \phi(W_1, W_2, Y_1, Y_2)$
  - ϕ is defined as the sharing rule; ϕ can be greater than Y, i.e. wage income can be transferred to spouse
  - The similar problem goes to person 2
- Finally, we can have noncooperative interaction (thus potentially inefficient): $\max U_1$ s.t. $C_1 = W_1(T - L_1) + Y_1$
  - Each individual takes the choice of the other individual as given

Public Goods and Household Production

- Family models become more interesting (and more complex) when we begin to add public goods (e.g children or housework or love that are enjoyed by all family members) which potentially come from household production and take both members' time as inputs

- In the unitary model, we can have $U(C, L_1, L_2, X)$ and a household production function $X = F(X_1, X_2)$
  - If the input here is time, it can be seen as another form of "leisure"
  - We can also allow other inputs that can be bought using wage

- We would then have two additional FOCs:
  $U_X(C, L_1, L_2, X) F_{X1} = \lambda W_1$
  $U_X(C, L_1, L_2, X) F_{X2} \geq \lambda W_2$

- Note the additional tradeoff:
  $U_{L1}(C, L_1, L_2, X) = U_X(C, L_1, L_2, X) F_{X1}$
  $U_{L2}(C, L_1, L_2, X) = U_X(C, L_1, L_2, X) F_{X2}$

- In the collective model, X enters both U_1 and U_2

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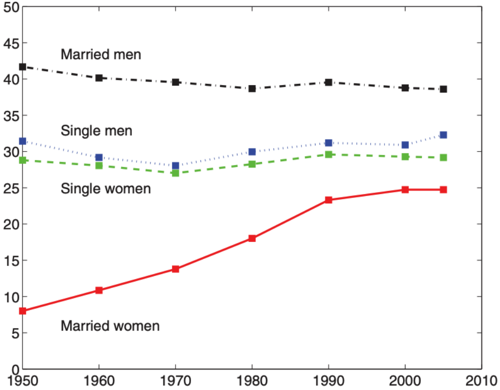
Labor Supply and Marriage



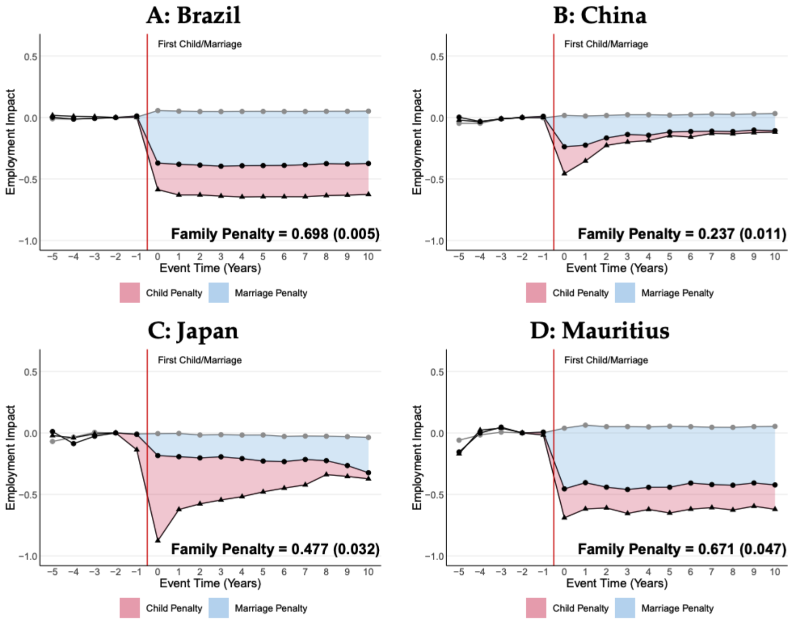
A. Market Hours of Work Per Week

Table 1.2
Civilian labor force participation rates of women aged 16 and older, classified by their marital status, in the United States.

	Single	Married
1900	45.9	5.6
1950	53.6	21.6
1988	67.7	56.7
2000	68.9	61.1
2010	63.3	61.0

Source: Ehrenberg and Smith (1994, table 6.1, p. 165) for 1900, 1950, and 1988 and Census Bureau for 2010.

Marriage and Child Penalty (see more countries)



What are the determinants of marriage/child penalty?

- Before marriage, an individual $i \in \{f, m\}$ with utility: $U = \max_{h_i} c(w_i h_i) + v(1 - h_i)$

- After marriage/child-birth, a unitary household with two members $i, j \in \{f, m\}$:
$$U = \max_{h_i, h_j, l_i, l_j, g} \left[c\left(w_i h_i + w_j h_j - pg\right) + \gamma_i v(l_i) + \gamma_j v(l_j) + f\left(\sigma_i(1 - h_i - l_i), \sigma_j(1 - h_j - l_j), \sigma_g g\right) n \right]$$

- Income effect (wh, γ)

- Substitution effect ▸ see the discussion of the corner solutions in a easier model
  - Wage difference (w)
  - Efficiency of childrearing (σ)
  - Note w and σ can capture potential differences in both absolute advantage & comparative advantage across the sexes
  - In collective models, we can also allow for sex differences in the responsibility of child rearing, which works similar to efficiency

- Many of these factors can be further due to social norms

- Market services or durable goods (g) with efficiency σ_g and price p

Explain Increased FLFP: Single Households

- Single females face the problem:
  $\max_{\{c_{fs}^m, c_{fs}^h, \ell_{fs}, \ell_{fs}^m, \ell_{fs}^h, k_{fs}\}} \mu \ln\left(c_{fs}^m\right) + \nu \ln\left(c_{fs}^h\right) + (1-\mu-\nu)\ln\left(\ell_{fs}\right)$
  s.t. $c_{fs}^m + pk_{fs} = w_f \ell_{fs}^m,\ c_{fs}^h = Ak_{fs}^{\theta}\left(\ell_{fs}^h\right)^{1-\theta},\ \ell_{fs} + \ell_{fs}^m + \ell_{fs}^h = 1$
  - k is durable goods for home production; p is the price or quality
  - A is a productivity factor; θ is the capital share in home production
  - $w_f \equiv (1-\tau)\,w$, where τ is a gender wage gap faced by a female due to e.g. discrimination; $w_m = w$

- Solution (Q: try solving the problem):
  $\ell_{fs}^m = \mu + \theta\nu,\ \ell_{fs}^h = (1-\theta)\nu,$

  $$c_{fs}^m = \mu w_f,\ c_{fs}^h = Ak_{fs}^{\theta}\left(\ell_{fs}^h\right)^{1-\theta},\ k_{fs} = \theta\nu w_f / p$$

  - ℓ_{fs}^m and ℓ_{fs}^h are independent of w, $1-\tau$ and A, p (Q: why?), and thus single women & men will work the same time under same μ, v, θ
  - In more general case, it depends on if home and market goods are substitutes or complements (more on next week); E.g. if complements, then an increase in A or a decline in p raises ℓ_{fs}^m

Explain Increased FLFP: Married Couples

- Assume collective and efficient households (cooperative via bargaining)
- $$\max \lambda_f \left[\mu \ln\left(c_{fp}^m\right) + \nu \ln\left(c_{fp}^h\right) + (1-\mu-\nu)\ln\left(\ell_{fp}\right)\right] + \lambda_m \left[\mu \ln\left(c_{mp}^m\right) + \nu \ln\left(c_{mp}^h\right) + (1-\mu-\nu)\ln\left(\ell_{mp}\right)\right],$$
  s.t.
  $$c_{fp}^m + c_{mp}^m + qk_p = w\ell_{fp}^m + w_f\ell_{mp}^m,\ c_{fp}^h + c_{mp}^h = Ak_p^\theta\left(\ell_{fp}^h + \ell_{mp}^h\right)^{1-\theta},$$
  $$\ell_{fp} + \ell_{fp}^m + \ell_{fp}^h = 1,\ \ell_{mp} + \ell_{mp}^m + \ell_{mp}^h = 1$$
  - As men's & women's hours are perfect substitutes in both home & market activities, solution to the problem is not interior in general
- In keeping with what is seen in the data, assume the case where $\ell_{mp}^h = 0$ and otherwise the solution is interior

Explain Increased FLFP: Married Couples

- Denote $w_p \equiv w + (1-\tau)\, w = w(2-\tau)$

- Solution (Q: try solving the problem):
  $$\ell_{fp}^m = 1 - \left[\lambda_f(1-\mu-\nu) + \nu(1-\theta)\right] \frac{w_p}{(1-\tau)\, w},$$
  $$\ell_{fp}^h = (1-\theta)\nu \frac{w_p}{(1-\tau)\, w},$$
  $$\ell_{mp}^m = 1 - \lambda_m(1-\mu-\nu)\frac{w_p}{w}$$
  - As for single households, changes in w, A, p, do not affect time uses
  - However, now reducing gender wage gap τ increases married women's working hours in the market (and decreases that for married men)

- Intuition: the wife's wage increase has wage effect dominated for herself as the income effect is shared with the husband under Pareto efficiency
  - (Q: what if only the husband is subject to an income tax and now the tax rate increases?)

Explanations for Increased FLFP in the Literature

- More productive opportunities for women (Galor and Weil, 1996) and reduced gender wage gap (Goldin, 1990; Jones et al., 2015)
- Home production through durable appliances (Greenwood et al., 2005) or marketization (Ngai et al., 2022)
- Introduction of the contraceptive pill and fertility changes (Goldin and Katz, 2002; Bailey, 2006) and medical advances of childbirth (Albanesi and Olivetti, 2016)
- Reductions in the cost of childcare (Attanasio et al., 2008)
- A change in women's bargaining power (Knowles, 2013)
- Cultural change, preference change, and learning (Fernández et al., 2004; Fogli and Veldkamp, 2011; Fernández, 2013)

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Gender and Education and Major

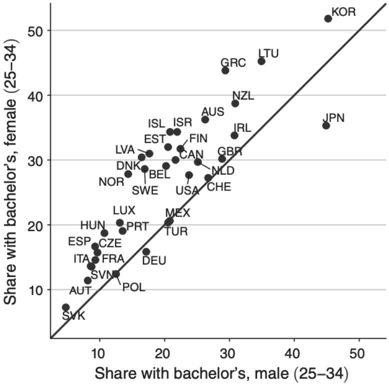


FIGURE 2. SHARES OF MEN AND WOMEN WITH AT LEAST A BACHELOR'S DEGREE

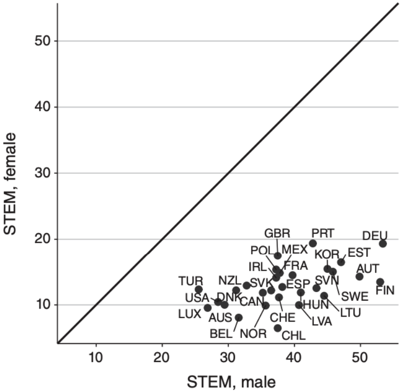
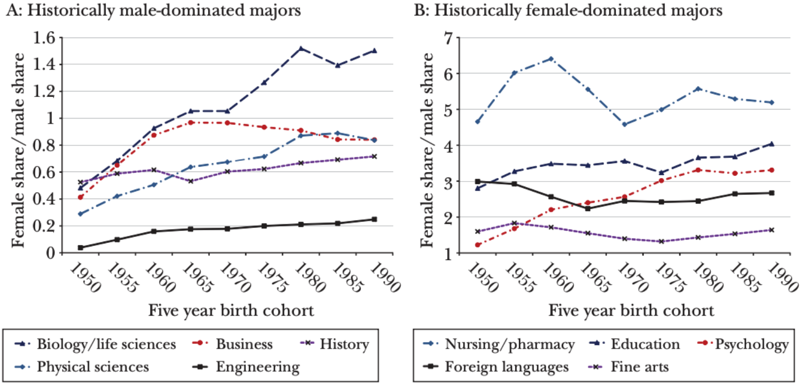


FIGURE 3. GENDER DIFFERENCES IN STEM EDUCATION (TERTIARY DEGREES)

Gender and Major - Cohort Trend

Figure 1

Gender Differences in Selected Majors by Birth Cohort

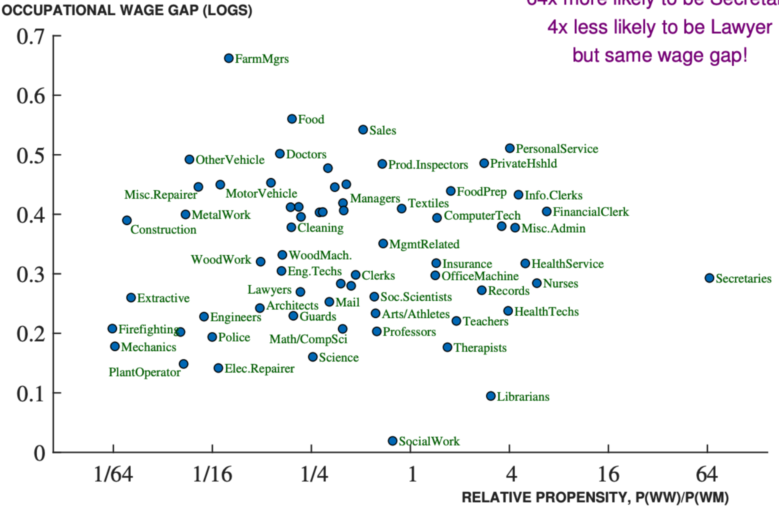


Source: Data from the 2014–2017 ACS and are restricted to those with at least a bachelor's degree. See text and the online Appendix for additional details.

Note: These figures plot the ratio of females to males within major category. The left panel shows trends for a set of majors where men outnumber women. The right panel shows trends for a set of majors where women outnumber men.

Gender and Occupation

Wage Gaps and Relative Propensities (Women in 1980)

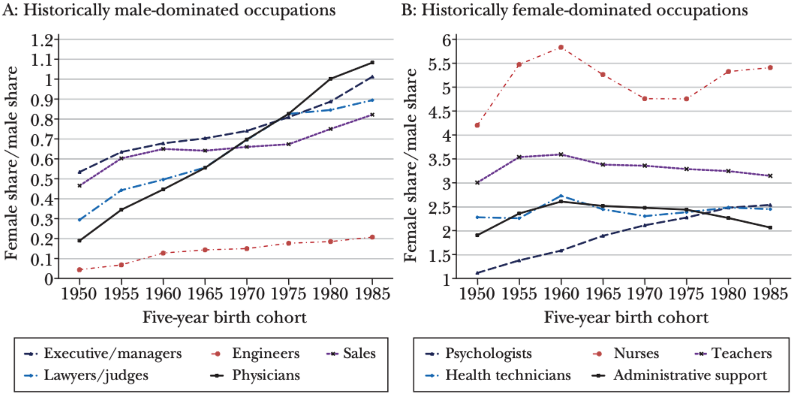


64x more likely to be Secretary
4x less likely to be Lawyer
but same wage gap!

Gender and Occupation - Cohort Trend

Figure 3

Gender Differences in Selected Occupations by Birth Cohort



Source: Data from the 2014–2017 ACS and are restricted to those with at least a bachelor's degree with non-missing occupation information. See text for additional details.
Note: These figures plot the ratio of females to males within broad occupation category. The left panel shows trends for a set of occupations where men outnumber women. The right panel shows trends for a set of occupations where women outnumber men.

Gender and Within-Occupation Wage Gap

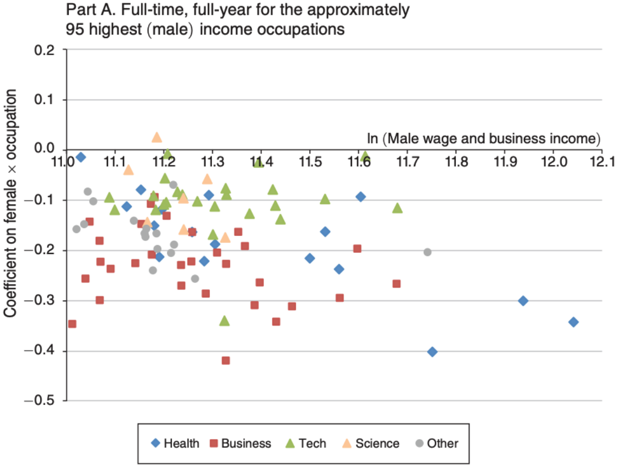


FIGURE 2A. GENDER PAY GAPS BY OCCUPATION: 2009 TO 2011

Notes: Sample consists of full-time, full-year individuals 25 to 64 years old excluding those in the military using trimmed annual earnings data (exceeding 1,400 hours × 0.5 × 2009 minimum wage). Regression contains age in a quartic, race, log hours, log weeks, education levels, census year, all occupations (469), and an interaction with female and occupation. Part A contains all full-time, full-year workers (2,603,968 observations); part B has those who graduated (BA) college (964,705 observations); part C has the group < 45 years old among those included in part A (1,333,013 observations). Each of the symbols in part A is an occupation

Goldin (2014): Nonlinear (convex) hours-wage relationship

- Some occupations exhibit linearity with respect to time worked whereas others exhibit nonlinearity (convexity)
- Gender differences in earnings across occupations substantially concern job flexibility and continuity (due to no perfect substitutes for a worker, i.e. turnover costs)
- Output/Wage: $\mathrm{Q} = \begin{cases} \lambda_i k_j & \text{if} \quad \lambda_i > \lambda_j^* \\ \lambda_i k_j \cdot (1 - \delta_j) & \text{if} \quad \lambda_i \leq \lambda_j^* \end{cases}$
  - $0 < \lambda_i \leq 1$ is the fraction of full-time employment worked by i
  - k_j is output per unit time in possition j
- Consider two positions exist such that $k_1 > k_2$, $\delta_1 > \delta_2$ and $k_1 (1 - \delta_1) < k_2 (1 - \delta_2)$
- The position with the highest return is also the one with the highest penalty with regard to reduced hours or more flexible employment (e.g. layers, C-suite, physicians vs pharmacists)

Erosa et al. (2022): Roy + Goldin

- A continuum of individuals, indexed by i, with utility:
  $\ln c_i - \phi_i \frac{(T+h_i)^{1+\gamma}}{1+\gamma}$ ▸ see the case of multi-member housholds
  - T reflects home production time
  - $\gamma > 0$ determines the curvature in disutility from work
  - ϕ_i is assumed to vary across individuals

- Assume two occupations and each i is endowed with a pair of occupational specific productivities (or human capital): (a_{i1}, a_{i2})

- Efficient labor units produced in occupation j: $e_j = a_j h^{1+\theta_j}$
  - Assume $\theta_1 > \theta_2 > 0$ so that occupation 1 features a greater nonconvexity

- Assume competitive market and same price for an efficiency unit of labor in either occupations

Erosa et al. (2022): Optimal labor choice

- Optimal labor supply decision can be solved in two stages: (i) chooses optimal hours given occupation (ii) chooses occupation

- In first stage, FOC: $\frac{1+\theta_j}{\phi_i} = h_j \left(T + h_j\right)^\gamma$ for $j = 1, 2$
  - h_j is independent of occupational productivity a_j (Q: why?)
  - h_j is decreasing in ϕ_i and T
  - $\theta_1 > \theta_2$ implies h_1 is greater than h_2

- In second stage, i chooses to work in occupation 1 if: $\ln\left(a_1 h_1^{1+\theta_1}\right) - \phi_i \frac{(T+h_1)^{1+\gamma}}{1+\gamma} > \ln\left(a_2 h_2^{1+\theta_2}\right) - \phi_i \frac{(T+h_2)^{1+\gamma}}{1+\gamma}$

- Rewrite: $\ln\left(\frac{a_1}{a_2}\right) > z(\phi) \equiv$ $-\left(1+\theta_1\right)\ln\left(h_1\right) + \left(1+\theta_2\right)\ln\left(h_2\right) + \phi\left[\frac{(T+h_1)^{1+\gamma}}{1+\gamma} - \frac{(T+h_2)^{1+\gamma}}{1+\gamma}\right]$
  - $z'(\phi) > 0$ given $\theta_1 > \theta_2$ and $h_1 > h_2$ (and Envelope Theorem)
  - Thus less comparative advantage required to choose occupation 2 as ϕ increases (Q: what if we also have T_i?)

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Appendix

Goldin (2006): Increased participation of women in US labor market since late 19th century

- 4 phases of the transition
  1. 1880s to 1920s: female workers were generally young and unmarried, exited the workforce at marriage
  2. 1930s to 1950s: increased demand for office & other clerical work, few remained employed after marriage partly due to marriage bars
  3. 1950s to 1970s: creation of part-time employment, elimination of marriage bars, But women were still largely secondary earners
  4. since later 1970s: revised expectations of future employment, further education, later marriage, rising earnings, varied occupations
- 3 important factors:
  1. ”Horizon” for human capital investment (intermittent or long-term)
  2. ”Decision making” in labor force decisions (”secondary worker” or own career)
  3. ”Identity” in job, occupation, profession, or career (individuality or social norm)
- Goldin summarized the US history, but these apply to other countries in general (e.g. Equal Employment Act in Japan in 1985)

Restriction: Income Pooling

- Only the total non-wage income $Y = Y_1 + Y_2$ matters
- This is a controversial assumption, often fails in empirical tests
- E.g. Lundberg et al. (1997) studies a ”natural experiment” that involved a redistribution of family income from men to women
  - A policy change in UK in the late 1970s transferred the child benefit scheme from a tax reduction in the father's paycheck into a direct child allowance to mothers, while largely holding constant total family income
  - They find a substantial increase in spending on women's & children's clothing, relative to men's clothing
  - Similar findings in some other studies
- E.g. Cesarini et al. (2017) finds lottery winners adjust labor supply more strongly than their spouses, independent on the winner's sex
- $\Rightarrow$ Thus policy targeting is important

Restriction: Slutsky symmetry

- Note in the last class with individual labor-leisure problem we have the Slutsky equation: $\frac{\partial L^m}{\partial w} = \frac{\partial L^h}{\partial w} + \frac{\partial L^m}{\partial Y} H$, where $\frac{\partial L^h}{\partial w} = \frac{\partial^2 Y(w,U)}{\partial w^2}$ since $Y_w(w, U) = -H^h(w, U)$
- In the unitary household model, we thus have $\frac{\partial L_1^h}{\partial w_2} = \frac{\partial^2 Y(\mathbf{w},U)}{\partial w_1 \partial w_2} = \frac{\partial^2 Y(\mathbf{w},U)}{\partial w_2 \partial w_1} = \frac{\partial L_2^h}{\partial w_1}$ where the second equation comes from the Young's theorem
- Thus we have the "Slutsky symmetry": $\frac{\partial L_1^m}{\partial w_2} - \frac{\partial L_1^m}{\partial Y} H_2 = \frac{\partial L_1^h}{\partial w_2} = \frac{\partial L_2^h}{\partial w_1} = \frac{\partial L_2^m}{\partial w_1} - \frac{\partial L_2^m}{\partial Y} H_1$
- This restriction on compensated price responses from unitary model is also typically rejected by empirical studies, see e.g. Browning and Meghir (1991); Deaton (1990)

A discrete choice model with uni-dimensional HC

- $\max_{j\in\{1,2\}} \mathbb{1}_{j=1} \underbrace{\left(\log\left(wh_i^{\psi_1}\cdot\kappa\right)+\nu_i\right)}_{\text{work}} + \mathbb{1}_{j=2} \underbrace{\log\left(h_i^{\psi_2}\right)}_{\text{housework}}$
- κ captures the systematic cost or taste for female working
- ν_i captures an idiosyncratic preference shock following a Gumbel distribution with scale parameter θ
- w is relative wage, i.e. normalized price for housework goods
- Assume human capital distribution: $\log(h) \sim F(h) = \mathcal{N}\left(\mu, \sigma^2\right)$
- Labor supply $l(h) = \frac{\left(w\kappa h^{\psi_1-\psi_2}\right)^\theta}{1+\left(w\kappa h^{\psi_1-\psi_2}\right)^\theta} \approx \left(w\kappa h^{\psi_1-\psi_2}\right)^\theta$
- Aggregate share of workers:
  $\pi = \int l(h)dF(h) = (w\kappa)^\theta \cdot \exp\left(\mu\left(\psi_1-\psi_2\right)\theta + \frac{\left(\left(\psi_1-\psi_2\right)\sigma\theta\right)^2}{2}\right)$

Child penalty in family framework

- Individual $i \in \{f, m\}$'s utility when married to their spouse j:
  $U_i\left(w_i, w_j\right) = \max_{0 \leq h_i \leq 1}\left[\delta_i w_i h_i + w_j h_j + \beta_i f\left(1 - h_i, 1 - h_j\right) n\right]$
  - each individual takes their partner's labor supply, h_j, as given
  - number of children, n, is assume to be exogenous
  - δ_i can be interpreted as the relative weight on consumption or career
  - β_i represents the value placed on the household good
  - note that we abstract from the leisure choices

- If there is no children (i.e. $n = 0$), both will work full-time (i.e. $h_m = 1$ and $h_f = 1$) as we abstract from leisure choice

- If $\delta_i = \delta_j = 1$ and $\beta_i = \beta_j$, the model is a unitary model: i.e. we can have a household with the problem
  $U\left(w_i, w_j\right) = \max_{h_i, h_j}\left[w_i h_i + w_j h_j + f\left(1 - h_i, 1 - h_j\right) n\right]$

Child penalty in unitary model

- Now consider a child is born and replace i, j with m, f
- Assuming a log linear production function of child rearing: $\ln\left[\alpha_m(1-h_m)+\alpha_f(1-h_f)\right]$ (thus perfect substitutes)
- First consider the case: $\alpha_m = \alpha_f = \alpha$
  - FOCs:
$$\begin{aligned} w_m =& \beta \frac{\alpha}{\alpha(1-h_m)+\alpha(1-h_f)} \\ w_f =& \beta \frac{\alpha}{\alpha(1-h_m)+\alpha(1-h_f)} \end{aligned}$$
  - $\Rightarrow$ If $w_m \neq w_f$ only one or none would hold for optimal solution
  - If $w_m > w_f$, and assuming $w_m > \beta$, men will devote all time to market work $h_m = 1$ (Q: what happens if $w_f < w_m < \beta$?)
  - Then if $w_f < \beta$, the wife spends all time in childrearing activities $h_f = 0$; Otherwise if $w_f > \beta$, $h_f^* = 1 - \frac{\beta}{w_f}$
- Then consider the case: $\alpha_f > \alpha_m \Rightarrow$ even if $w_f > w_m$, we may observe women reducing hours or dropping out of labor force

Child penalty in non-cooperative collective model

- Now each spouse solves their own optimization problem, taking the other's action h_j as given: $\max_{0 \leq h_i \leq 1} U_i\left(h_i, h_j\right) = \delta_i w_i h_i + w_j h_j + \beta_i \ln\left(\alpha_i\left(1 - h_i\right) + \alpha_j\left(1 - h_j\right)\right)$
- Optimal labor supply within the household after the arrival of the child is determined by the ratio: $\frac{\beta\alpha}{\delta w}$
  - The spouse with lower ratio will devote more time to market work
- Assuming that at least one spouse works full-time, the analysis is similar to that of the unitary model, where the spouse with the higher $\frac{\beta\alpha}{\delta w}$ ratio will work in the market if $w_i > \frac{\beta_i \alpha_i}{\delta_i}$, and his/her labor supply will be given by $h_i^* = 1 - \frac{\beta_i \alpha_i}{\delta_i w_i}$
- In sum, the model suggests that compared to men, women are more likely to work fewer hours or exit the labor force if
  - Same preferences & productivities, but the wife faces a lower wage
  - Wives face higher wages, but they are either more productive at home $(\alpha_f > \alpha_m)$, value the household public good more $(\beta_f > \beta_m)$, or suffer a utility penalty from working in the market $(\delta_f < \delta_m)$

Erosa et al. (2022): Multi-Member Households

- $U(c_m, c_f, h_m, h_f) = u_m(c_m, h_m) + u_f(c_f, h_f)$, where
  $u_g(c_g, h_g) = \ln c_g - \phi_g \frac{(T_g + h_g)^{1+\gamma}}{1+\gamma}$ for $g = m, f$
  - Gender differences in T_g can reflect gender differences in responsibilities at home

- Optimal household allocations are the solution to
  $$\max \left\{ \ln c_m + \ln c_f - \phi_m \frac{(T_m + [I_1^m h_{m1} + I_2^m h_{m2}])^{1+\gamma}}{1+\gamma} - \phi_f \frac{\left(T_f + \left[I_1^f h_{f1} + I_2^f h_{f2}\right]\right)^{1+\gamma}}{1+\gamma} \right\}$$
  $$\text{s.t. } c_m + c_f = \left\{ \sum_{j_m=1}^{2} I_{j_m}^m a_{mj_m} g_1(h_{mj_m}) + \sum_{j_f=1}^{2} I_{j_f}^f a_{fj_f} g_2(h_{fj_f}) \right\}$$
  - indicator functions $I_{j_g}^g$ takes 1 if g chooses occupation $j_g = 1, 2$
  - optimal allocation of consumption implies $c_m = c_f = c$

- Optimal choice of hours $\bar{h}$ conditional on occupational choices:
  $$\frac{a_{mj_m}(1+\theta_{j_m}) h_{mj_m}^{\theta_{j_m}}}{\left(a_{mj_m} h_{mj_m}^{1+\theta_{j_m}} + a_{fj_f} h_{fj_f}^{1+\theta_j}\right)/2} = \phi_m (T_m + h_{mj_m})^{\gamma},$$
  $$\frac{a_{fj_f}(1+\theta_{j_f}) h_{fj_f}^{\theta_j}}{\left(a_{mj_m} h_{mj_m}^{1+\theta_{j_f}} + a_{fj_f} h_{fj_f}^{1+\theta_{j_f}}\right)/2} = \phi_f (T_f + h_{fj_f})^{\gamma}$$