

# Labor Supply: Income and Substitution Effects

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# Roadmap

1. Introduction
2. Some Empirical Facts
3. Before the Theory
4. Labor-Leisure Choice Model
5. Math Derivation of Slutsky Equation \*
6. Some Applications
7. Math behind Applications \*

# Labor Supply Decisions

- ▷ People decide
  - ▷ **whether to work or not** (*extensive margin*)
  - ▷ **how many hours to work** (*intensive margin*)
  - ▷ how hard to work
  - ▷ when to quit a job
  - ▷ which skills to acquire
  - ▷ which occupations to enter
- ▷ What factors affect these decisions?
  - ▷ E.g. consider you are currently working on a part-time job and then (i) the wage becomes double or (ii) you win a lottery

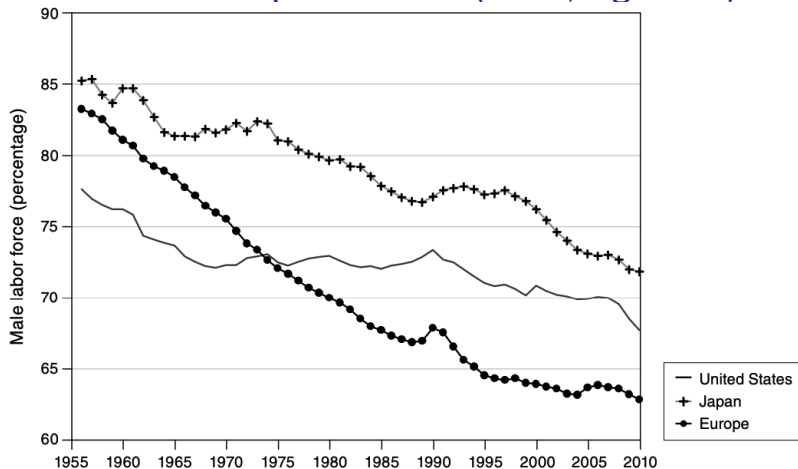
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# Measures of labor supply

- ▷ **Extensive margin:** labor force participation rate
  - ▷ Labor force (LF) = employed (E) + unemployed (U)
  - ▷ LFP Rate =  $LF / \text{working age population}$
- ▷ **Intensive margin:** working hour per worker

# Labor Force Participation Rate (Male, Age 15+)



**FIGURE 1.3**

The evolution in civilian labor force participation rates of men in the United States, Europe, and Japan for persons 15 years of age and older, 1956–2010.

Source: OECD Annual Labor Force Statistics.

*(What can cause the declines here?)*

# Labor Force Participation Rate (Male by Age)

**Table 6.2**

**Labor Force Participation Rates for Males in the United States, by Age, 1900–2008 (percentage)**

| Year | Age Groups |       |       |       |       |         |
|------|------------|-------|-------|-------|-------|---------|
|      | 14–19      | 16–19 | 20–24 | 25–44 | 45–64 | Over 65 |
| 1900 | 61.1       |       | 91.7  | 96.3  | 93.3  | 68.3    |
| 1910 | 56.2       |       | 91.1  | 96.6  | 93.6  | 58.1    |
| 1920 | 52.6       |       | 90.9  | 97.1  | 93.8  | 60.1    |
| 1930 | 41.1       |       | 89.9  | 97.5  | 94.1  | 58.3    |
| 1940 | 34.4       |       | 88.0  | 95.0  | 88.7  | 41.5    |
| 1950 | 39.9       | 63.2  | 82.8  | 92.8  | 87.9  | 41.6    |
| 1960 | 38.1       | 56.1  | 86.1  | 95.2  | 89.0  | 30.6    |
| 1970 | 35.8       | 56.1  | 80.9  | 94.4  | 87.3  | 25.0    |
| 1980 |            | 60.5  | 85.9  | 95.4  | 82.2  | 19.0    |
| 1990 |            | 55.7  | 84.4  | 94.8  | 80.5  | 16.3    |
| 2000 |            | 52.8  | 82.6  | 93.0  | 80.4  | 17.7    |
| 2008 |            | 40.1  | 78.7  | 91.9  | 81.4  | 21.5    |

Sources: 1900–1950: Clarence D. Long, *The Labor Force under Changing Income and Employment* (Princeton, N.J.: Princeton University Press, 1958), Table A-2.

1960: U.S. Department of Commerce, Bureau of the Census, *Census of Population, 1960: Employment Status*, Subject Reports PC(2)–6A, Table 1.

1970: U.S. Department of Commerce, Bureau of the Census, *Census of Population, 1970: Employment Status and Work Experience*, Subject Reports PC(2)–6A, Table 1.

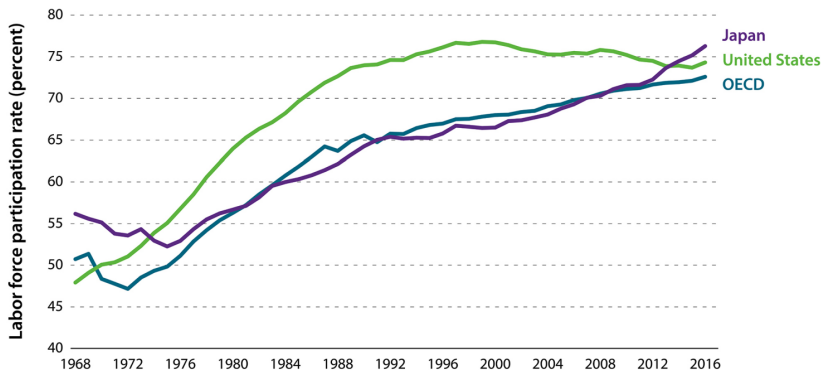
1980–2008: U.S. Census Bureau, *2010 Statistical Abstract*, Section 12 (Table 575), <http://www.census.gov/compendia/statab/2010edition.html>.

*(Men are starting their work lives later and ending them earlier than before.)*

# Labor Force Participation Rate (Female, Prime-age)

FIGURE 1.

Labor Force Participation of Prime-age Women from 1968–2016, by Country



Source: OECD Labour Force Statistics.

Note: Prime-age indicates 25–54.

*(Women had a very different trend compared to men! More next week!)*



# Working Hours per Worker: Trend (Boppart and Krusell, 2020)

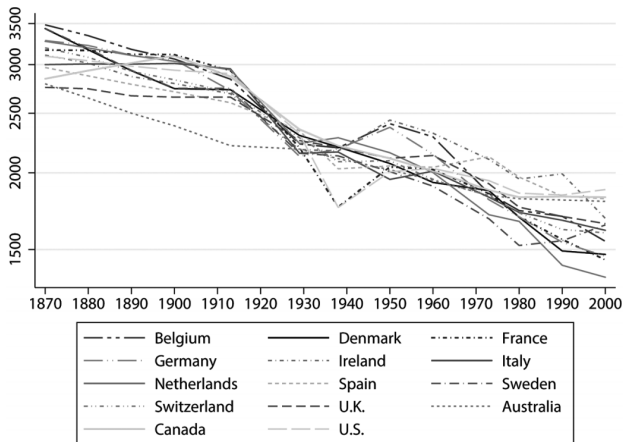


FIG. 1.—Hours worked per worker. The figure shows data for the following countries: Belgium, Denmark, France, Germany, Ireland, Italy, the Netherlands, Spain, Sweden, Switzerland, the United Kingdom, Australia, Canada, and the United States. The scale is logarithmic, which suggests that hours fall at roughly 0.57% per year. Source: Huberman and Minns (2007). Maddison (2001) shows a similar systematic decline in hours per capita. A color version of this figure is available online.

*(What cause the declines here?)*

# Working Hours per Worker: Cross-country (Bick et al., 2018)

Panel B. Hours per worker

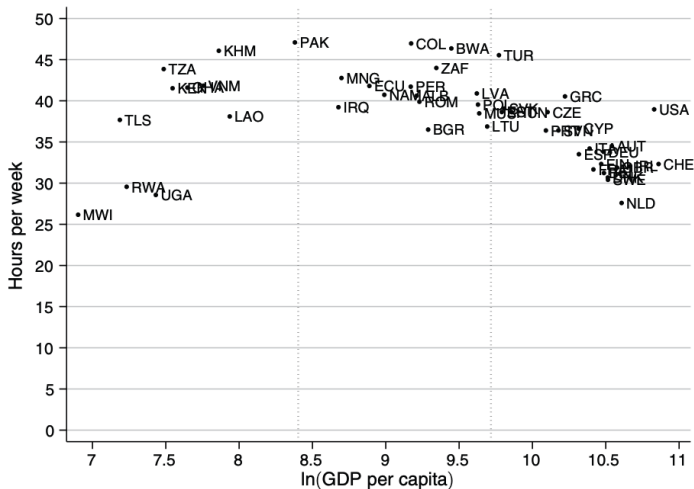


FIGURE 3. EXTENSIVE AND INTENSIVE MARGINS IN CORE COUNTRIES

*(What cause the inverted-U shape here?)*

# Working Hours per Person: US vs OECD (Rogerson, 2024)

*Table 1*

## **Hours of Work per Person Relative to the United States**

| Considerably below the<br>US level (<0.75) | Moderately below the<br>US level (0.75, 0.85) | Slightly below the<br>US level (0.85, 0.95) | At or above the<br>US level (>0.95) |
|--|---|---|-------------------------------------|
| Italy (0.69)                               | Finland (0.77)                                | UK (0.85)                                   | Canada (0.96)                       |
| France (0.70)                              | Austria (0.79)                                | Sweden (0.90)                               | Australia (0.98)                    |
| Belgium (0.72)                             | Norway (0.80)                                 | Ireland (0.91)                              | United States (1.00)                |
| Greece (0.73)                              | Netherlands (0.82)                            | Japan (0.91)                                | New Zealand (1.07)                  |
| Denmark (0.74)                             | Portugal (0.85)                               | Switzerland (0.93)                          | Korea (1.12)                        |
| Germany (0.74)                             |   |   |                                     |
| Spain (0.75)                               |   |   |                                     |

*Source:* Author's calculation using data from OECD (2024a, c).

*Note:* Details of the calculation are in the online Appendix. Table shows average for 2015–2019.

*(Why do Europeans and Japanese work less than Americans?)*

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# What are the Potential Drivers of Labor Supply?

- ▷ We focus on "economics" factors b.c. in our economics models agents behave under "economics" incentives
  - ▷ Wage; Income; Wealth
  - ▷ Leisure activities; Housework
  - ▷ Taxes; Welfare policies/programs
- ▷ General mechanisms are more useful and "scientific" than just saying
  - ▷ "Europeans are much lazier than Americans"
  - ▷ "Japaneses have the culture of working hard"
- ▷ Furthermore,
  - ▷ culture is often formed due to economics incentives
  - ▷ it's in fact not difficult to incorporate culture factors into econ models

# Labor Supply in Roy Framework

- ▷ Consider a setting of either work or home production
- ▷ Two choices:
  - ▷ Work in labor market, receive  $wh^m$
  - ▷ Work at home and produce  $ph^h$
- ▷ A person  $i$  works in labor market if

$$wh_i^m > ph_i^h$$

- ▷ People who are relatively more productive in the market will work
- ▷ Total labor supply, which sums all individual choices, depends on
  - ▷ relative price  $w/p$
  - ▷ joint distribution of human capital  $F(h^m, h^h)$
- ▷ Here, only extensive margin of labor supply is considered

# Setting of A Labor-Leisure Model

- ▷ Agent:
  - ▷ Individuals of working age
- ▷ Decision/Choices:
  - ▷ How many **hours for work/leisure** per day
  - ▷ Note this choice nests both extensive (0) and intensive margin
- ▷ Time:
  - ▷ Simple static choice
- ▷ Equilibrium:
  - ▷ Partial equilibrium where wage is taken as given

# Can Workers Choose Working Hours?

- ▷ Don't employers set the hours of work? (e.g. **Ford in 1926**)
- ▷ Workers can
  - ▷ choose part-time vs full-time
  - ▷ select industries/occupations/firms with different working hours
  - ▷ shirk during their working time
  - ▷ initiate **labor movements**
- ▷ So the argument is that employer requirements on work hours will reflect workers' preferences, esp. in the long-run
  - ▷ What's behind cultural and political movements can be thus utility maximization
- ▷ But firms (labor demand side) surely have some power in setting working hours
  - ▷ Over business cycles (**Kudoh et al., 2019**)
  - ▷ Across industries/occupations (e.g. law or IB firms) (**Bertrand et al., 2010**)



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## At a High Level

- ▶ The "neoclassical theory of labor supply": focus on individual choice
- ▶ An application of consumer theory: choose between two goods (consumption and leisure)
  - ▶ The tricky part: here agents simultaneously choose consumption and "income" by choosing working hours
- ▶ Thus more close to what you have learned in your microeconomics class

# Setting

- ▷ The agent has preference, i.e. a **utility function**  $U(C, L)$ 
  - ▷  $C$  is consumption of goods and services (w/ normalized price  $p = 1$ )
  - ▷  $L$  is leisure
  - ▷ Assume  $U(\cdot, \cdot)$  is a strictly increasing and strictly concave (intuition: decreasing marginal return)
- ▷ The agent has two endowments:
  - ▷ Disposable time  $T$ : 24 or 16 or 12 hours
  - ▷ Non-wage income  $Y$ : can be 0 or even negative (debt)
- ▷ The agent maximize utility by choosing  $L$  or working time  $H$ 
  - ▷  $L + H = T$  thus choosing one pins down another
  - ▷ Static optimization as no multiple periods and no savings
- ▷ Assume wage  $w$  is taken as given and does not depend on  $H$

# Optimization

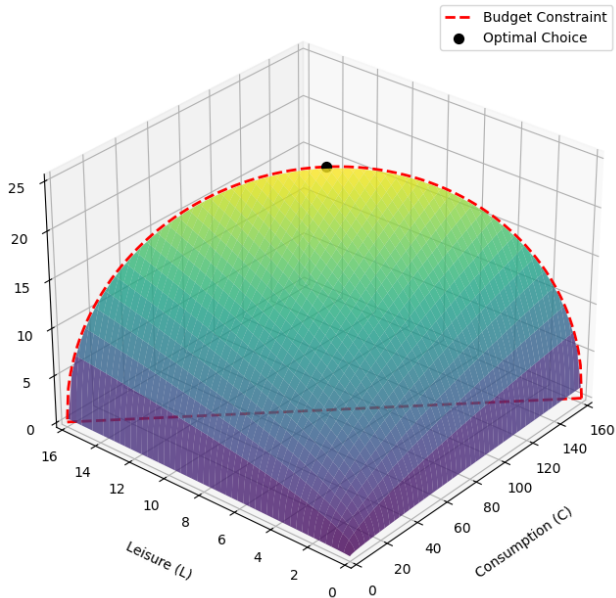
- ▷ Problem:  $\max_{C,L} U(C, L)$  subject to  $C = w(T - L) + Y$
- ▷ Note the budget constraint can be also written as  $Tw + Y = Lw + C$ 
  - ▷  $Tw + Y$  can be referred to as "full income"
  - ▷ The price (opportunity cost) for  $L$  is  $w$
  - ▷ A rise in  $w$  increases both full income and cost of leisure
- ▷ Alternatively:  $\max_{C,H} V(C, H) = U(C, T - H)$  s.t.  $C = wH + Y$ 
  - ▷ Can also regard  $H$  in  $V()$  as a negative term, i.e. disutility

# Derivation

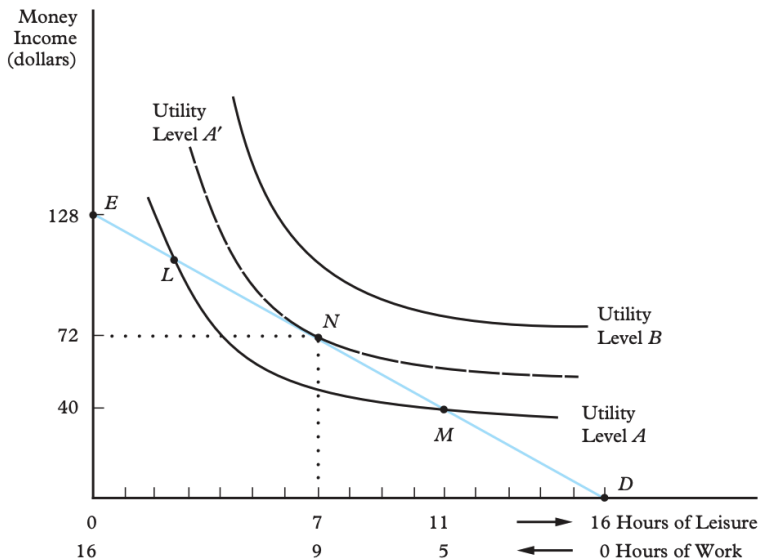
- ▷  $\max_{C,L} U(C, L)$  s.t.  $C = w(T - L) + Y$
- ▷ Lagrangian:  $\mathcal{L} = U(C, L) - \lambda (C - w(T - L) - Y)$
- ▷ Assume an interior optimum, the First Order Conditions (FOCs):  
 $\mathcal{L}_C = U_C - \lambda = 0$   
 $\mathcal{L}_L = U_L - \lambda w = 0$   
 $\mathcal{L}_\lambda = C - w(T - L) - Y = 0$
- ▷ **Tradeoff:**  $U_L(C^*, L^*) = wU_C(C^*, L^*)$ 
  - ▷ Note  $U_L/U_C$  is the marginal rate of substitution (MRS), which equates to  $w$ , the relative price
- ▷ **Marshallian (Uncompensated) Demand functions:**  
 $L = L^m(w, Y)$   
 $C = C^m(w, Y)$
- ▷ Lagrange multiplier:  $\lambda = U_C = \lambda^m(w, Y)$ 
  - ▷ Interpreted as marginal utility or "shadow price" of income

# Visualize Optimization *(see the code)*

Labor-Leisure Optimization Problem



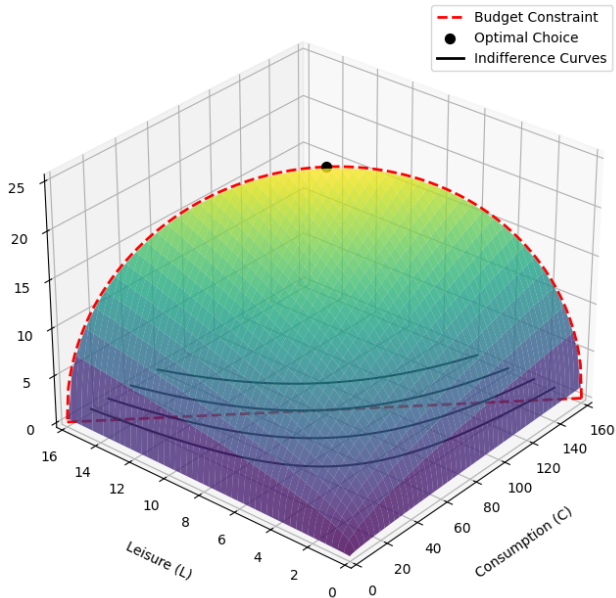
# Indifference Curves and Budget Constraint Curve



*(The indifference curves bending outward (convex to origin) comes from our concavity assumption; But why we don't want it bending inward?)*

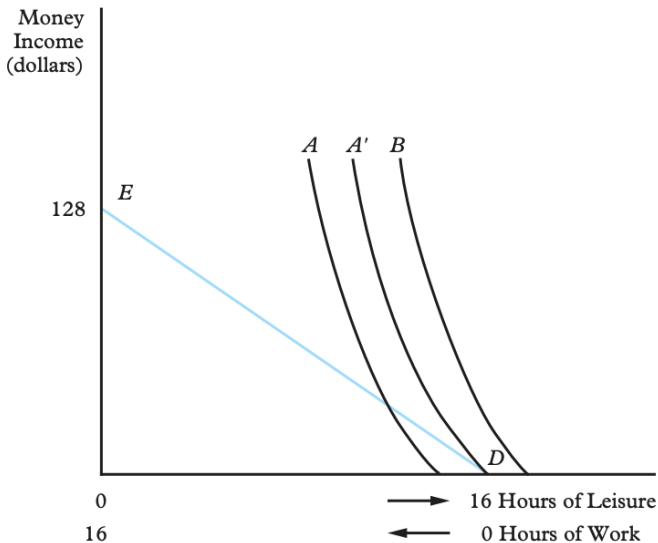
# IC and BC in 3D Plot

Labor-Leisure Optimization Problem





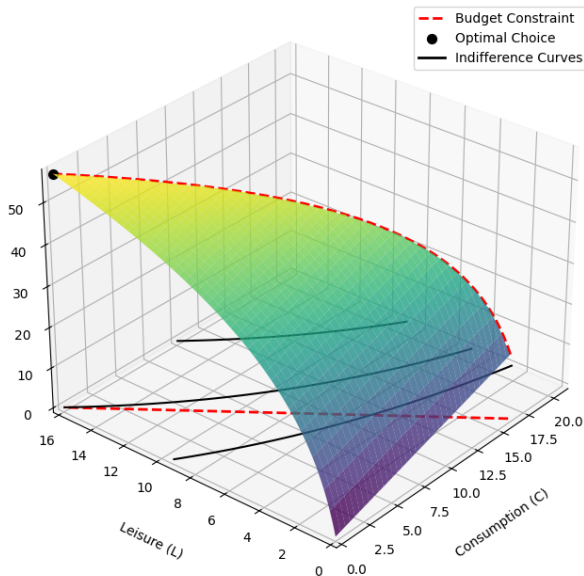
# Not-Work is A Corner Solution ( $U_L > wU_C$ )



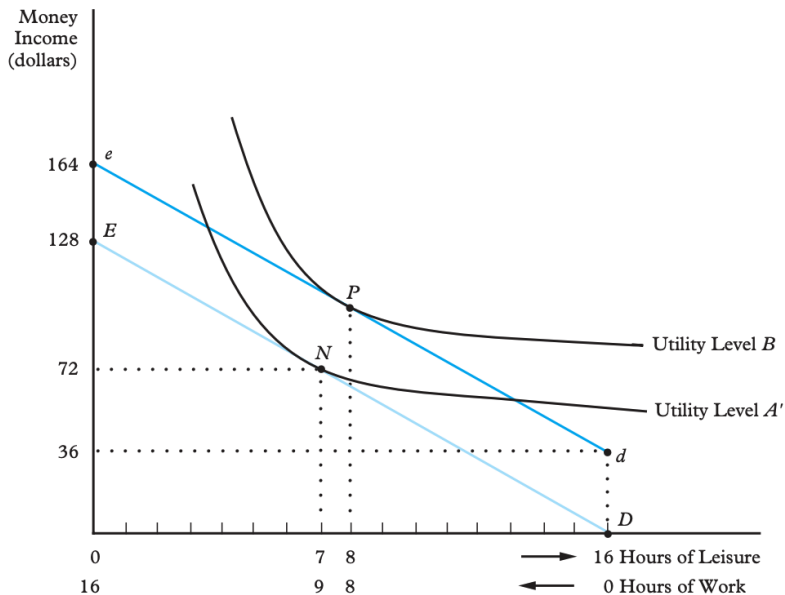
(We can define a "reservation wage"  $\underline{w}$  by  $\underline{w} = U_L(Y, T) / U_C(Y, T)$ , i.e. the wage that is just low enough to induce the agent to supply a tiny unit of labor)

# Corner Solution in 3D Plot

Labor-Leisure Optimization with Quasi-Linear Utility Function (Corner Solution)



# Income Effect (An Increase in $Y$ )



# Income Effect in Math

- ▷ Income Effect:

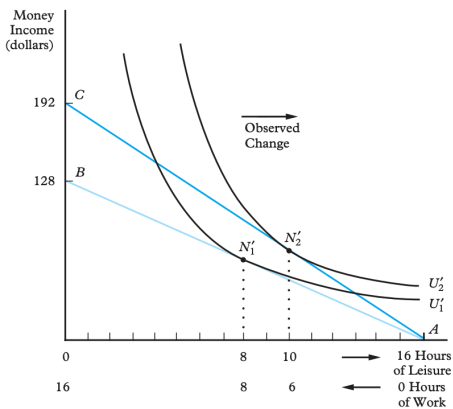
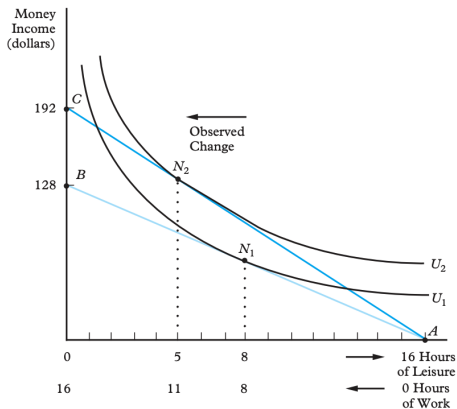
$$\frac{\partial L^m}{\partial Y}$$

- ▷ Income elasticity of leisure demand

$$\varepsilon_{L,Y} = \frac{\partial L^m}{\partial Y} \frac{Y}{L} = \frac{\partial \ln L^m}{\partial \ln Y}$$

- ▷ Leisure is generally regarded as a **normal good**, i.e.  $0 < \varepsilon_{L,Y} \leq 1$ 
  - ▷ **Inferior good** if  $\varepsilon_{L,Y} \leq 0$ ; **Luxury good** if  $\varepsilon_{L,Y} > 1$
  - ▷ This means  $\frac{\partial L^m}{\partial Y} > 0$
- ▷ The sign depends on the utility function used
  - ▷ E.g. **CD utility functions**,  $U(C, L) = C^\alpha L^\beta$  with  $\alpha, \beta > 0$  and  $\alpha + \beta \leq 1$ , imply both  $C$  and  $L$  are normal goods
- ▷ Cases of observing income effect:
  - ▷ lottery; bequest; government cash transfer

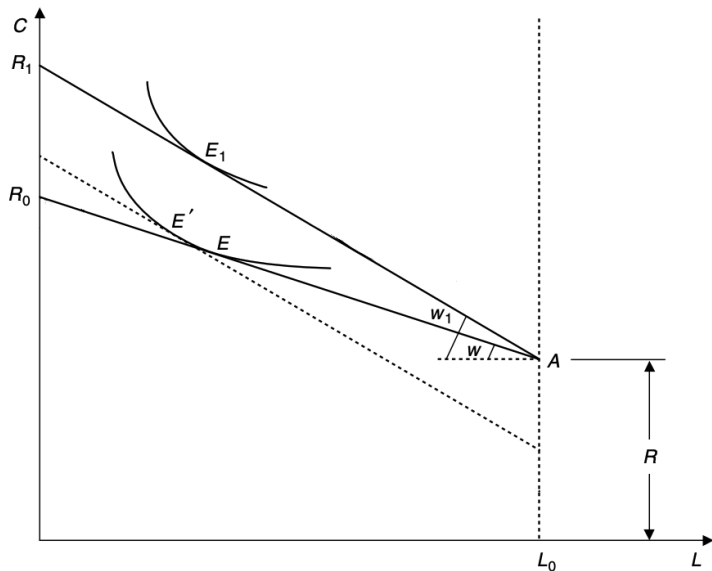
# Wage Effect (An Increase in $w$ )



*(The result, again, depends on the shape of the indifference curve—i.e. the utility function!)*

*(It turns out that this wage change nests two effects: substitution and income effects!)*

## Income + Substitution Effect (An Increase in $w$ )



Step 1: Maintain the initial utility but twist to new wage ( $E \rightarrow E'$ ; substitution effect);

Step 2: Shift to new budget constraint and find optimal level ( $E' \rightarrow E_1$ ; income effect)

# Wage Effect in Math

- ▷ Slutsky equation:

$$\frac{\partial L^m}{\partial w} = \underbrace{\frac{\partial L^h}{\partial w} \Big|_U}_{\text{Substitution Effect (-)}} + \underbrace{\frac{\partial L^m}{\partial Y} H^h \Big|_w}_{\text{Income Effect (+)}}$$

- ▷ Elasticity form:

$$\underbrace{\varepsilon_{L,w}}_{\text{Uncompensated Elasticity}} = \underbrace{\varepsilon_{L,w}^c}_{\text{Compensated Elasticity}} + \varepsilon_{L,Y} \frac{wH}{Y}$$

- ▷ The net effect depends on the relative size of two effects
- ▷ Estimated results in the microeconomics literature are small:
  - ▷  $\varepsilon_{H,w} \in [-0.1, 0.2]$  and  $\varepsilon_{H,w}^c \in [0.1, 0.3]$
- ▷ Cases of observing wage (price) effects:
  - ▷ income tax; minimum wage
  - ▷ "Price" changes in leisure activities or home production

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# The Dual Problem

*(This problem helps to conduct the calculation in Step 1)*

- ▶ The **dual problem** is to minimize the expenditure to achieve some utility  $U$ :

$$Y(w, U) = \min_{C, L} C - w(T - L)$$

s.t.  $U(C, L) \geq U$

- ▶ Referred as "excess expenditure function"
- ▶ Here  $Y$  is no longer a given parameter in the budget constraint but the value of the objective function
- ▶  $\mathcal{L} = C - w(T - L) - \lambda (U(C, L) - U)$
- ▶ **Hicksian (Compensated) Demand** functions:  
 $C = C^h(w, U)$   
 $L = L^h(w, U)$
- ▶ Expenditure function:  $Y(w, U) = C^h(w, U) - w(T - L^h(w, U))$

# Derive Slutsky Equation

- ▶ Sheppard's lemma:  $Y_w(w, U) = -(T - L^h(w, U)) = -H^h(w, U)$   
(take derivative of the expenditure function and use *Envelop theorem*)
- ▶ The Hicksian and Marshallian demand functions for leisure are related to each other:  $L^h(w, U) \equiv L^m(w, Y(w, U))$
- ▶ Differentiating:

$$\frac{\partial L^h}{\partial w} = \frac{\partial L^m}{\partial w} + \frac{\partial L^m}{\partial Y} \frac{\partial Y}{\partial w}$$

- ▶ Slutsky equation:

$$\frac{\partial L^m}{\partial w} = \underbrace{\frac{\partial L^h}{\partial w}}_{\text{Substitution Effect (-)}} + \underbrace{\frac{\partial L^m}{\partial Y} H^h}_{\text{Income Effect (+)}}$$

(Be careful that we have rearranged the equation!)

# What Do $\frac{\partial L^h}{\partial w}$ and $\frac{\partial L^m}{\partial Y}$ Depend On?

- ▷ Slutsky equation in utility terms: *(see next slide for derivation)*

$$\frac{\partial L}{\partial w} = \frac{U_C - (U_{LC} - wU_{CC})(T - L)}{U_{LL} + w^2U_{CC} - 2wU_{LC}}$$

- ▷ The denominator is the SOC of the problem and thus negative given concavity *(see in two slides)*
- ▷ Thus  $\frac{\partial L}{\partial w} \propto -U_C + (U_{LC} - wU_{CC})H$
- ▷  $-U_C$  captures the substitution effect, which is proportional to the marginal utility of consumption
- ▷  $(U_{LC} - wU_{CC})H$  captures the income effect, which depends on the cross-derivative and the concavity of the utility function in consumption
  - ▷ Now you can see why for Quasi-linear utility functions the income effect is 0 ( $U_{LC} = 0$ ;  $U_{CC} = 0$ )

# Derive Slutsky Equation from Utility Function

- ▶ Total differentiating  $\frac{U_L}{U_C} = w$  with respect to  $w$
- ▶  $\frac{U_C \frac{\partial U_L}{\partial w} - U_L \frac{\partial U_C}{\partial w}}{U_C^2} = 1 \Rightarrow \frac{\partial U_L}{\partial w} - \frac{U_L}{U_C} \frac{\partial U_C}{\partial w} = U_C \Rightarrow \frac{\partial U_L}{\partial w} - w \frac{\partial U_C}{\partial w} = U_C$
- ▶  $\Rightarrow U_{LL} \frac{\partial L}{\partial w} + U_{LC} \frac{\partial C}{\partial w} - w(U_{CC} \frac{\partial C}{\partial w} + U_{LC} \frac{\partial L}{\partial w}) = U_C$
- ▶ From  $C = (T - L)w + Y \Rightarrow \frac{\partial C}{\partial w} = T - L - w \frac{\partial L}{\partial w}$
- ▶  $\Rightarrow U_{LL} \frac{\partial L}{\partial w} + (U_{LC} - wU_{CC})(T - L) + w^2 U_{CC} \frac{\partial L}{\partial w} - 2wU_{LC} \frac{\partial L}{\partial w} = U_C$
- ▶  $\Rightarrow \frac{\partial L}{\partial w} = \frac{U_C - (U_{LC} - wU_{CC})(T - L)}{U_{LL} + w^2 U_{CC} - 2wU_{LC}}$
- ▶ Note that you can totally differentiate w.r.t.  $Y$  to get  $\frac{\partial L}{\partial Y}$ , which directly gives you the formula of income effect!  
(In fact, the more general way to do all the derivations is to total differentiate FOCs w.r.t  $w$  and  $Y$  in the matrix form and then to solve the system)

## Second Order Condition

- ▷ As we are dealing with constrained optimization, we examine how the objective function changes according to a vector of budget-neutral variations:  $(dC, dL) = (-w, 1)dL$  (because we need  $dC = -wdL$  for budget not changing)

- ▷ The second-order effect of such a variation is

$$(-w, 1) \begin{bmatrix} U_{CC} & U_{CL} \\ U_{LC} & U_{LL} \end{bmatrix} \begin{bmatrix} -w \\ 1 \end{bmatrix} = w^2 U_{CC} - 2w U_{CL} + U_{LL} < 0$$

- ▷  $\begin{bmatrix} U_{CC} & U_{CL} \\ U_{LC} & U_{LL} \end{bmatrix}$  is the Hessian matrix of the utility function
- ▷ Quadratic form  $(v'Qv)$  here tells about the steepness or curvature of the specific path following our budget-neutral variations
- ▷ We can prove this inequality holds with strictly quasi-concave (s.q.c) utility function (see last year's slide)
- ▷ In fact, the concavity assumed already ensures the Hessian matrix to be negative semi-definite, i.e. the quadratic form to be negative for all non-zero vectors  $v$
- ▷ That's why FOCs are both necessary and sufficient to characterize an interior "preference maximal" with s.q.c!

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# Why Declined Working Hours?

*Boppart and Krusell (2020):  $w \uparrow$  and income effect dominated!*

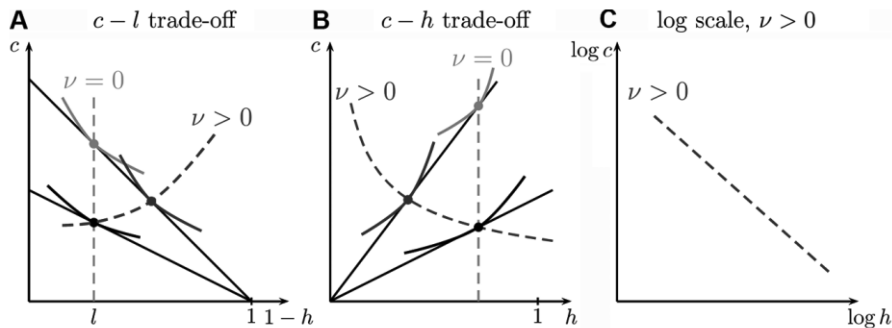
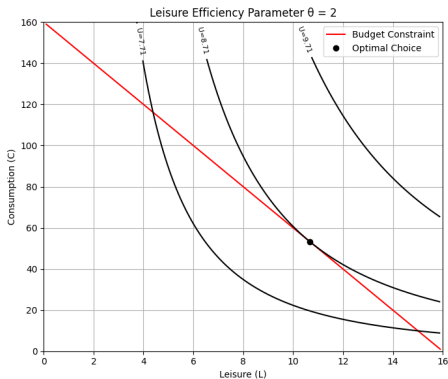
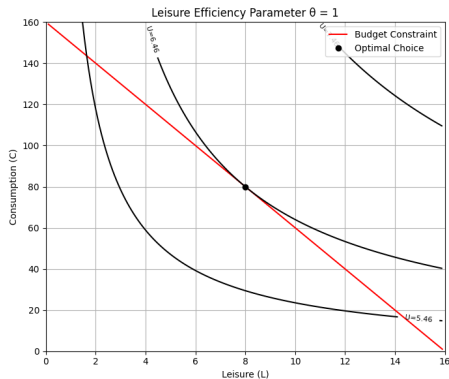


FIG. 5.—Consumption-leisure trade-off. The figure panels abstract from unearned income. A color version of this figure is available online.

(Their utility function:  $u(c, h) = \frac{(c \cdot v(hc^{v/(1-\nu)}))^{1-\sigma} - 1}{1-\sigma}$ ; with  $\sigma > 1$ ,  $\nu > 0$ ;  $c^{\frac{\nu}{1-\nu}}$  captures a stronger income effect: an added "penalty" to working (since  $\nu$  is a decreasing function); They thus support the *Keynes' speculation*: people will work 15-hour week in the future!)

# Why Declined Labor Supply for Young Men?

*Aguiar et al. (2021): better recreational compute use and gaming increases efficiency of leisure time*

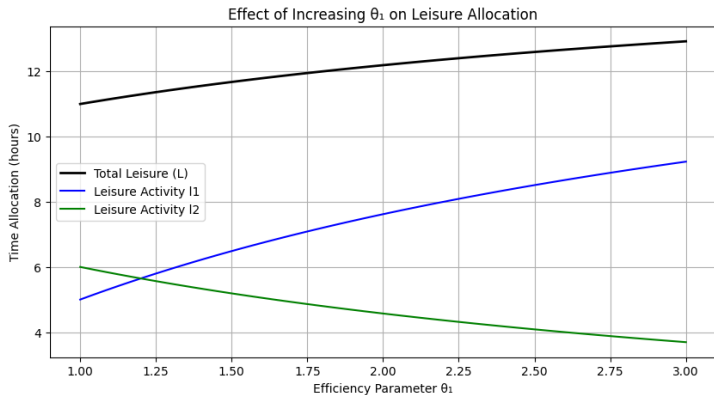


(Here consider an extremely simple case:  $U(C, L) = \log(C) + \theta \log(L)$  and recreational technology increases  $\theta$ ; Intuition: increased efficiency is similar to reduce price, generating substitutions effects)



# Why Declined Labor Supply for Young Men?

*Aguiar et al. (2021): split entire leisure time into various leisure activities; find recreational computing is a "leisure luxury" for younger men*

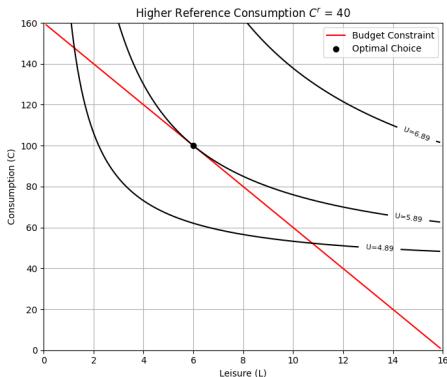
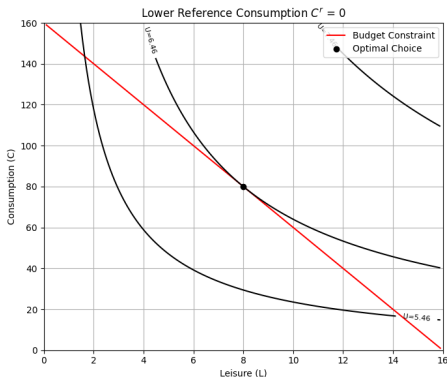


(Extend our simple utility function:  $U(C, l_1, l_2) = \log(C) + \theta_1 \log(l_1) + \theta_2 \log(l_2)$ , where  $L = l_1 + l_2$ ; Intuition: increased efficiency like reduced price generates "leisure income effect" and "leisure substitution effect")

# Bring Sociology into Economics

**Thorsten Veblen:** consumption is motivated by a desire for social standing, and other social classes strive to emulate the leisure class

**Bowles and Park (2005)** brings this idea into the labor-leisure framework to see how emulated consumption affect labor supply:  $U = (C - C^r, L)$



(Intuition: an increase in reference consumption  $C^r$  increases marginal utility of consumption, requiring more consumption and less leisure to balance the tradeoff)

# Roadmap

1. Introduction
2. Some Empirical Facts
3. Before the Theory
4. Labor-Leisure Choice Model
5. Math Derivation of Slutsky Equation \*
6. Some Applications
7. Math behind Applications \*

# What Utility Functions Have No Income Effect?

- ▷ **Quasi-linear utility:**  $U(C, L) = C + V(L)$
- ▷ E.g.  $U(C, L) = C + \frac{L^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$ 
  - ▷ The optimality condition:  $L^* = w^\varepsilon$
  - ▷ Thus the optimal choice of  $L^*$  is not a function of income (or more accurately, not a function of consumption  $c$ )
- ▷ In fact, with any quasilinear utility, we have  $\frac{\partial L^m}{\partial Y} = 0$
- ▷ Intuition:
  - ▷ marginal utility of leisure is not a function of consumption
  - ▷ marginal utility of consumption is constant
- ▷ Further,  $\varepsilon_{L,w}^U = \varepsilon_{L,w}^C = \partial \log L / \partial \log w = \varepsilon$ 
  - ▷ Thus wage elasticity of labor supply is a constant (purely through the substitution effect)

# What Utility Functions Have No Wage Effect?

- ▶ Macroeconomists like to use utility functions with a form close to  $u(c, l) = cv(l)$ , where  $v(\cdot)$  satisfies the usual conditions
- ▶ Recall FOC:  $u_c w = u_l$
- ▶  $\Rightarrow v(l)w = cv'(l)$
- ▶ If there is no wealth, i.e.  $y = 0$ , then  $c = wh = w(1 - l)$
- ▶  $\Rightarrow v(l)w = w(1 - l)v'(l) \Rightarrow v(l) = (1 - l)v'(l)$ , i.e.  $l^*$  does not depend on  $w$  as income and substitution effects cancel out
- ▶ If  $y > 0$ ,  $v(l) = (1 - l + y/w)v'(l)$ ,
  - ▶ i.e. an increase in wage will reduce leisure as substitution effect dominates (intuition: the income effect is now smaller with  $y > 0$ )
  - ▶ In macro models,  $y$  and  $w$  will always grow in the same speed, so  $y/w$  is a constant and  $l^*$  will be stationary

# What Utility Functions Have Declined Working Hour?

- ▶ MaCurdy (1981):  $u(c, h) = \frac{c^{1-\sigma}-1}{1-\sigma} - \psi \frac{h^{1+1/\theta}}{1+1/\theta}$  ( $\sigma, \theta \geq 0$ )
- ▶ FOC:  $w c^{-\sigma} = \psi h^{1/\theta}$
- ▶  $\Rightarrow h^* = \psi^{\frac{-1}{\sigma+1/\theta}} w^{\frac{1-\sigma}{\sigma+1/\theta}}$
- ▶ If  $\sigma > 1$ ,  $h^*$  decreases with  $w$  increase, i.e. income effect dominates substitution effect
- ▶ If  $\sigma = 1$ , it return backs to previous case of perfect offsetting
  - ▶ To see this: first obtain  $\frac{c^{1-\sigma}-1}{1-\sigma} = \log(c)$  when  $\sigma = 1$ ; then take exponential of  $u$  to obtain a form of  $cv(I)$
- ▶ General form studied in King et al. (1988) and Boppart and Krusell (2020)

# What Explain Declined Labor Supply for Young Men?

- ▶ Aguiar et al. (2021) suggests better recreational computing and gaming
- ▶ Agent now chooses between multiple leisure activities in addition to the work-leisure tradeoff: e.g.

$$\max_{c, \{\ell_1, \dots, \ell_I\}, H} U(c, v(\ell; \theta))$$

$$\text{s.t. } c \leq wH \text{ and } \sum_{i=1}^I \ell_i + H \leq 1, \text{ where } v(\ell; \theta) = \sum_{i=1}^I \frac{(\theta_i \ell_i)^{1-(1/\eta_i)}}{1-(1/\eta_i)}$$

- ▶ Opportunity cost for each leisure activity is not only wage, but also the utility from choosing other activities
- ▶ They estimate this leisure demand system and find r.c.g. is a "leisure luxury" specially for younger men
  - ▶ (1% increase in leisure time associated with about a 2.5% increase in r.c.g. time)
- ▶ While the key idea is very simple, the model derivation and estimation are nontrivial (see last year's slides)

# Add Emulated Consumption into the Framework

- ▷ Assume  $u = u(c^o, h) = u[(wh - vc^r), h]$ 
  - ▷  $c^r \equiv w^r h^r + y$  is the consumption level of some rich reference group
  - ▷  $v$  measure the intensity of the relevant social comparisons
- ▷ Note that  $vc^r$  here plays the same role as a negative non-wage income  $y$ , i.e. a debt
- ▷ If the utility function is Quasi-linear in  $h$ , e.g.  $u = \ln c^o - \delta h$ 
  - ▷ Optimal solution:  $h^* = 1/\delta + vc^r/w$
- ▷  $dh^*/dc^r \propto -v(u_{c^o h} + wu_{c^o c^o})$  is positive (same is  $dh^*/dv$ )
- ▷ With many income groups each of which takes the next richest group as its reference group, an increase in consumption by the top rich generates a downward cascade of Veblen effects



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